Multiuser Signature Quantization With Tree-Structured Codebook in DS-CDMA

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***Abstract*—We consider a signature quantization scheme for a group of users in a reverse-link direct sequence (DS)-code division multiple access (CDMA). Assuming perfect channel knowledge, a receiver selects the set of signatures that maximizes an average signal-to-interference plus noise ratio (SINR) from a random vector quantization (RVQ) codebook, which consists of independent isotropically distributed unit-norm vectors. The quantized signatures are relayed from the receiver to users via noiseless rate-limited feedback channels. Previously, we have proposed to organize entries of RVQ codebook into a tree structure (TS) to speed up a search for the optimal entry. Here we extend the TS scheme for a *multiuser* signature quantization. Numerical results show that for a given performance, a TS-RVQ codebook can be an order of magnitude less complex than an RVQ codebook.**

I. Introduction

We can improve a user performance in a direct sequence (DS)-code division multiple access (CDMA) by optimizing the user's signature sequence [1]. A receiver, which can estimate an interference covariance matrix, can compute the optimal signature for the user. The optimal signature that minimizes an interference power is the eigenvector of the interference-plus-noise covariance matrix corresponding to the smallest eigenvalue. The user then obtains the optimal signature from the receiver via a feedback channel. However, a feedback channel in practice has a very limited rate and thus, the signature computed by the receiver needs to be quantized. Some of our previous work [2]–[4] and references therein have considered signature quantization in CDMA. Solutions to this signature quantization problem can also be applied in a multiantenna system where a spatial signature, which consists of transmit antenna weights, is quantized [4]–[8].

In [2], [9], we have proposed a Random Vector Quantization (RVQ) codebook, which consists of independent isotropically distributed unit-norm vectors and we have showed that RVQ is optimal over all codebooks in a large system limit in which processing gain, number of interfering users, and number of available feedback bits tend to infinity with fixed ratios. The large system limit has been shown to predict the performance of a finite-size system very well [9]. Furthermore, RVQ performs close to the optimal codebook for a finite-size system [5], [8]. However, RVQ and most of the codebooks proposed in literature require an exhaustive search to locate the optimal signature. With B feedback bits available, the codebook contains 2^B entries. Thus, the search complexity increases exponentially with a number of feedback bits and becomes an issue for a large B. Other simpler quantization schemes [7], [10] have been proposed. In [7], RVQ entries were organized into a tree and thus, the search complexity increases linearly with B. A codebook with entries consisting of only QAM symbols was considered in [10] and it was shown that a number of searches required also grows linearly with B. In [2], [9], a number of coefficients to quantize is reduced by projecting the signature vector onto a lower dimensional subspace and the coefficients are then scalar quantized. The complexity of this scheme is much less than the vector quantization, but the performance also suffers greatly.

Here we extend the TS-RVQ scheme proposed in [7] to *multiple users*. Even though there have been much work on signature quantization for a single user, signature quantization for group of users has not been paid much attention. This problem should interest a service operator, who would like to differentiate quality of service for a certain group of users. For practical reason, we consider a frequency-selective channel with path loss. Compared with an RVQ, TS-RVQ may give a worse performance for a given number of feedback bits. However, we show that search complexity of TS-RVQ can be an order of magnitude less than that of RVQ.

II. System Model

We consider a discrete-time reverse-link synchronous DS-CDMA with processing gain N. We assume that K_a users adapt their signature sequences while the rest of users, K_b, do not adapt signature sequences and hence, the *total* number of users $K = K_a + K_b$. The $N \times 1$ received vector is given by

$$\boldsymbol{r} = \sum_{k=1}^{K_a} \sqrt{A_k} \boldsymbol{H}_k \boldsymbol{s}_k b_k + \sum_{i=1}^{K_b} \sqrt{A_i} \boldsymbol{H}_i \boldsymbol{p}_i b_i + \boldsymbol{n} \qquad (1)$$

where $\sqrt{A_k}$ is an amplitude of user k, $\boldsymbol{H}_k$ is an $N \times N$ channel matrix for user k, $\boldsymbol{s}_k$ is an $N \times 1$ signature vector for adapting user k, $\boldsymbol{p}_i$ is an $N \times 1$ signature vector for interfering user i, b_k is a transmitted symbol for user k, $\boldsymbol{n}$ is an additive white Gaussian noise with zero mean and covariance $\sigma_n^2 \boldsymbol{I}$, and $\boldsymbol{I}$ is an identity matrix.

For an ideal non-fading channel, $\boldsymbol{H}_k = \boldsymbol{I}$ for all k. For a frequency-selective fading channel, assuming that each user

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traverses L fading paths, we have

$$\boldsymbol{H}_k = \begin{bmatrix} h_{k,1} & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & h_{k,1} & & \vdots & 0 & & \vdots \\ h_{k,L} & \vdots & \ddots & 0 & \vdots & & 0 \\ 0 & h_{k,L} & & h_{k,1} & 0 & \dots & 0 \\ \vdots & 0 & \ddots & \vdots & h_{k,1} & & 0 \\ 0 & \vdots & & h_{k,L} & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 0 & h_{k,L} & \dots & h_{k,1} \end{bmatrix} \quad (2)$$

where fading gains for user k, $h_{k,1}, \dots, h_{k,L}$ are independent complex Gaussian random variables with zero mean and variance $E|h_{k,1}|^2, \dots, E|h_{k,L}|^2$, respectively. For a flat fading channel (L = 1), $\boldsymbol{H}_k = h_{k,1}\boldsymbol{I}$. Besides fading, a signal transmitted through medium also undergoes attenuation. It is well known that signal attenuation or path loss can be modeled as a function of distance [11], [12]. Thus, a signal power for user k is given by

$$A_{k,\text{dB}}(d) = A_{k,\text{dB}}(d_o) + 10\alpha \log\left(\frac{d}{d_o}\right) + X \quad (3)$$

where $A_{k,\text{dB}}(d)$ is A_k in dB at distance d from a signal source, d_o is a reference distance, α is a path loss exponent, and X is a zero mean log-normal random variable whose unit is in dB.

We assume that the receiver uses a single-user matched filter given by

$$\boldsymbol{c}_k = \frac{\boldsymbol{H}_k \boldsymbol{s}_k}{|\boldsymbol{H}_k \boldsymbol{s}_k|} \quad (4)$$

to detect user k's transmitted symbol. An average output signal-to-interference plus noise ratio (SINR) over all K_a adapting users is given by

$$\text{SINR}(\boldsymbol{S}) = \frac{\sum_{k=1}^{K_a} A_k (\boldsymbol{s}_k^\dagger \boldsymbol{H}_k^\dagger \boldsymbol{H}_k \boldsymbol{s}_k)^2}{I + \sigma_n^2 \sum_{k=1}^{K_a} (\boldsymbol{s}_k^\dagger \boldsymbol{H}_k^\dagger \boldsymbol{H}_k \boldsymbol{s}_k)} \quad (5)$$

where an interference power

$$I = \sum_{k=1}^{K_a} \sum_{l=1, l\neq k}^{K_a} A_l (\boldsymbol{s}_k^\dagger \boldsymbol{H}_k^\dagger \boldsymbol{H}_l \boldsymbol{s}_l)^2 + \sum_{k=1}^{K_a} \sum_{i=1}^{K_b} A_i (\boldsymbol{s}_k^\dagger \boldsymbol{H}_k^\dagger \boldsymbol{H}_i \boldsymbol{p}_i)^2 \quad (6)$$

and an $N \times K_a$ matrix containing signatures for K_a users

$$\boldsymbol{S} = [\boldsymbol{s}_1 \quad \boldsymbol{s}_2 \ \cdots \ \boldsymbol{s}_{K_a}]. \quad (7)$$

We note that SINR depends on a signature set $\{\boldsymbol{s}_k\}$ or $\boldsymbol{S}$ and thus, we would like to maximize this average SINR over a signature set $\{\boldsymbol{s}_k\}$. If the feedback channel between the receiver to users has unlimited rate, the optimized signature set can be fed back to users unquantized. The optimization problem with unlimited feedback has been solved in [1]. With limited number of feedback bits, the signature needs to be quantized. In what follows, we describe the proposed signature quantization scheme and show associated performance.

III. Tree-Structured RVQ

An RVQ codebook was first proposed by [2] to quantize a single signature and the codebook contains independent isotropically distributed vectors. This codebook design was motivated by the fact that the optimal signature is the eigenvector of an interference covariance matrix, which is isotropically distributed. Reference [9] generalizes the codebook design in [2] to quantize multiple signatures. For multiple users, an RVQ codebook consists of matrices whose columns are signature vectors for optimizing users and are independent isotropically distributed, and is denoted by

$$\mathcal{V} = \{\boldsymbol{V}_1, \boldsymbol{V}_2, \dots, \boldsymbol{V}_n\}. \quad (8)$$

The receiver selects the signature matrix from the codebook that maximizes the average SINR in (5)

$$\boldsymbol{S} = \arg\max_{\mathcal{V}} \text{SINR}(\boldsymbol{V}_j) \quad (9)$$

for given channel and set of interfering signatures. We describe the exact steps in Algorithm 1. For RVQ, we must compute

Algorithm 1 Search algorithm for the optimal set of signatures in RVQ

SINR($\boldsymbol{S}$) = 0
for $j = 1 : n$ **do**
  Compute SINR($\boldsymbol{V}_j$).
  if SINR($\boldsymbol{V}_j$) $\geq$ SINR($\boldsymbol{S}$) **then**
    $\boldsymbol{S} = \boldsymbol{V}_j$
    SINR($\boldsymbol{S}$) = SINR($\boldsymbol{V}_j$)
  end if
end for
return the optimal set of signatures $\boldsymbol{S}$.

SINR for every signature matrices in the codebook to locate the optimal signature set. Thus, the search complexity grows linearly with number of entries in the codebook, which is a function of available feedback bits. To reduce the complexity, a tree-structured (TS) RVQ scheme was proposed by [7]. The codebook entries are organized into a tree and thus, the search complexity grows logarithmically with number of entries.

Here we modify the previously proposed TS codebook for a multiuser signature quantization. We start with an RVQ codebook with independent isotropically distributed *vectors*, which is denoted by

$$\mathcal{V}_T = \{\boldsymbol{v}_1, \quad \boldsymbol{v}_2, \ \cdots \ , \boldsymbol{v}_{nK_a}\}. \quad (10)$$

We note that for a subsequent performance comparison, we set a number of vector entries in $\mathcal{V}_T$ equal to a total number of column vectors in RVQ codebook $\mathcal{V}$. To build a tree, we apply a generalized Lloyd Algorithm (GLA) [7], [13]. First, we find two vector centroids $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ that minimize a sum of Euclidean distances between all vector entries to the centroids,

$$d = \sum_{v_j \in \mathcal{V}_{T,1}} \|\boldsymbol{c}_1 - \boldsymbol{v}_j\| + \sum_{v_k \in \mathcal{V}_{T,2}} \|\boldsymbol{c}_2 - \boldsymbol{v}_k\| \quad (11)$$

where the two quantization regions or clusters are given by

$$\mathcal{V}_{T,1} = \{\boldsymbol{v}_j \in \mathcal{V}_T : \|\boldsymbol{c}_1 - \boldsymbol{v}_j\| \leq \|\boldsymbol{c}_2 - \boldsymbol{v}_j\|\} \quad (12)$$

and

$$\mathcal{V}_{T,2} = \mathcal{V}_T \backslash \mathcal{V}_{T,1}. \tag{13}$$

We iterate between computing the sum distance d and finding the clusters until d converges. In the end, we obtain two child nodes, which associate with $\mathcal{V}_{T,1}$ and $\mathcal{V}_{T,2}$, respectively. Then, we move to the child node associated with $\mathcal{V}_{T,1}$ or $\mathcal{V}_{T,2}$ and repeat the process until we obtain all leaf nodes, which contain only one vector entry. With this algorithm, we eventually create a binary tree, which may not be balanced. Building a tree requires an extensive computing power especially for a large codebook. Fortunately, it can be done offline and thus, computation complexity should not be a problem.

With TS codebook, we can search for the optimal set of signatures by using Algorithm 2. For signature of user 1, we start at the root node and transverse the tree by comparing SINR($\boldsymbol{c}_1$) and SINR($\boldsymbol{c}_2$) where $\boldsymbol{c}_1$ and $\boldsymbol{c}_2$ are centroids of the vector sets of the left and right child nodes, respectively. We then move to the node with the signature that gives larger SINR and repeat these steps until we reach the leaf node. For subsequent users, we start at the root node and follow the same steps, but we also account for signatures we obtain from earlier iterations (see steps 5 and 6). The complexity here relies

Algorithm 2 Search Algorithm for TS-RVQ

1: $\boldsymbol{S} = [\quad]$.
2: **for** $i = 1 : K_a$ **do**
3: Start at the root node of a TS-RVQ codebook with $\boldsymbol{S}$.
4: **while** Not a leaf node. **do**
5: $\boldsymbol{V}_L = [\boldsymbol{S} \quad \boldsymbol{c}_1]$.
6: $\boldsymbol{V}_R = [\boldsymbol{S} \quad \boldsymbol{c}_2]$.
7: **if** SINR($\boldsymbol{V}_L$) $\geq$ SINR($\boldsymbol{V}_R$) **then**
8: Move to the left child node.
9: $\boldsymbol{T} = \boldsymbol{V}_L$.
10: **else**
11: Move to the right child node.
12: $\boldsymbol{T} = \boldsymbol{V}_R$.
13: **end if**
14: **end while**
15: $\boldsymbol{S} = \boldsymbol{T}$.
16: **end for**
17: **return** $\boldsymbol{S}$

largely on SINR computation in step 7. For each iteration in step 2, $\boldsymbol{S}$ remains the same. Hence, part of SINR computation in step 7 that is associated with $\boldsymbol{S}$ may only be determined once. Only SINR computations related to column vector $\boldsymbol{c}_1$ or $\boldsymbol{c}_2$ for $\boldsymbol{V}_L$ and $\boldsymbol{V}_R$, respectively, need to be obtained at every node the algorithm visits. Thus, computing SINR($\boldsymbol{V}_L$) or SINR($\boldsymbol{V}_R$) take only $1/K_a$ of the complexity for computing SINR($\boldsymbol{V}_j$) in (5) for an RVQ codebook. We remark that the total complexity of the TS-RVQ will depend on a number of nodes or the tree's depth.

Next we show the performance of the proposed quantization scheme and compare with that of the original RVQ codebook.

IV. Numerical Results

Here we show some simulation results to illustrate the performance of RVQ and TS-RVQ for both ideal and fading reverse-link channels. Fig. 1 shows the SINR for an ideal channel with normalized feedback bit, which equals a total number of feedback bits per processing gain (B/N) for both RVQ and TS-RVQ. We set $N = 10$, $K_a = 2$, $K_b = 2$, and background signal-to-noise ratio (SNR) $= 1/\sigma_n^2 = 10$ dB. Thus, for $B/N = 1$, $B = 10$ and a number of column vectors in both codebooks equals 2×2^{10}. For larger parameters, a number of codebook entries will be enormous and hence, we will need a very powerful computer to perform simulations. However, the results shown with a relatively small system are still useful for predicting the performance of a larger system.

As expected, both feedback schemes give better performance as amount of feedback increases. For smaller feedback, TS-RVQ performs a bit better while for larger feedback, RVQ is expected to perform better with its exhaustive search. With 1 feedback bit per processing gain, the performance of RVQ is 1.5 dB better than that of TS-RVQ.

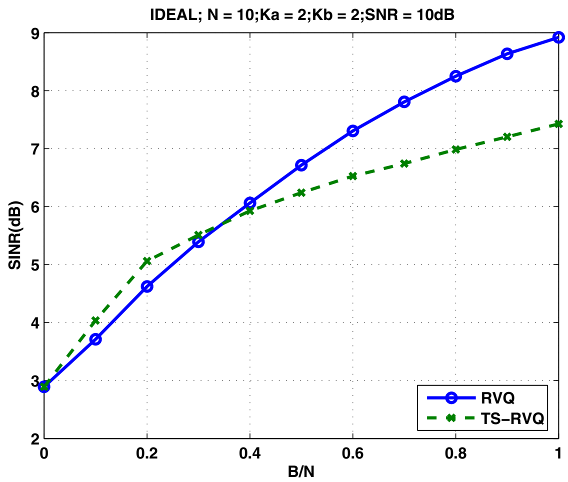

Fig. 1. Shown is an average SINR versus a normalized feedback bit for an ideal non-fading channel. Performance of RVQ and TS-RVQ are compared with $N = 10$, $K_a = 2$, $K_b = 2$, and SNR $= 10$ dB.

In Fig. 2, we show a number of SINR computations required to locate the optimal signature set against SINR for both RVQ and TS-RVQ. The parameters and settings are the same as those in previous figure. For small SINR, a search complexity of both schemes are comparable. As SINR increases (or as a number of feedback bits increases), the search complexity for RVQ increases exponentially while that for TS-RVQ increases linearly. We see that for SINR = 7.5 dB, TS-RVQ needs 20 computations while RVQ needs more than 80 computations. We note that the gap between complexity of both schemes will only widen as SINR increases. Thus, for a given performance, TS-RVQ offers a simpler quantization scheme than RVQ does.

Figs. 3 and 4 show SINR performance of RVQ and TS-RVQ and complexity of both schemes for a frequency-selective fading channel with path loss. We set a number of fading paths $L = 2$ for all users, path loss exponent $\alpha = 3$, $N = 10$, $K_a = 2$, and $K_b = 4$. From the figures, we observe similar performance trends to those for a non-fading channel. With half a feedback bit per processing gain, the performance of the

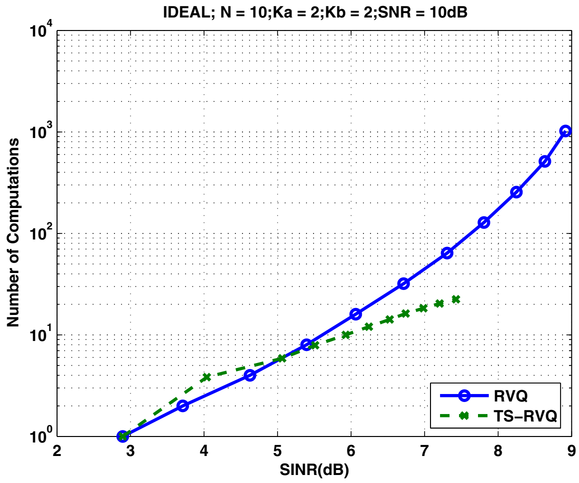

Fig. 2. Shown is a number of SINR computations versus a normalized feedback bit for an ideal non-fading channel with $N = 10$, $K_a = 2$, $K_b = 2$ and SNR = 10 dB.

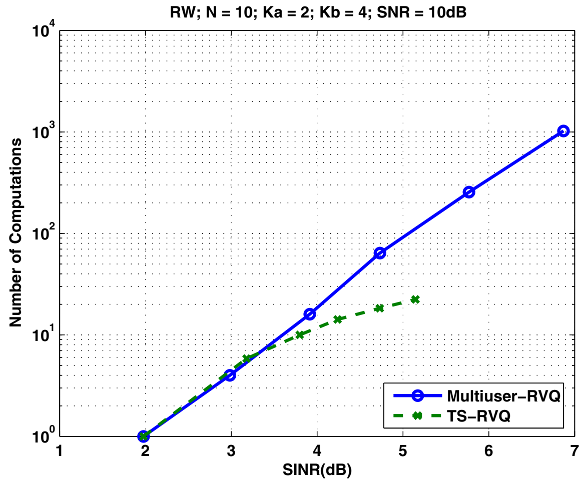

Fig. 4. A number of SINR computation needed for RVQ and TS-RVQ is shown with SINR for a frequency-selective fading channel with path loss. The system parameters are $N = 10$, $K_a = 3$, $K_b = 4$, $L = 3$, $\alpha = 3$ and SNR = 10 dB.

two schemes are approximately the same. However, RVQ is an order of magnitude more complex than TS-RVQ for SINR at 5 dB.

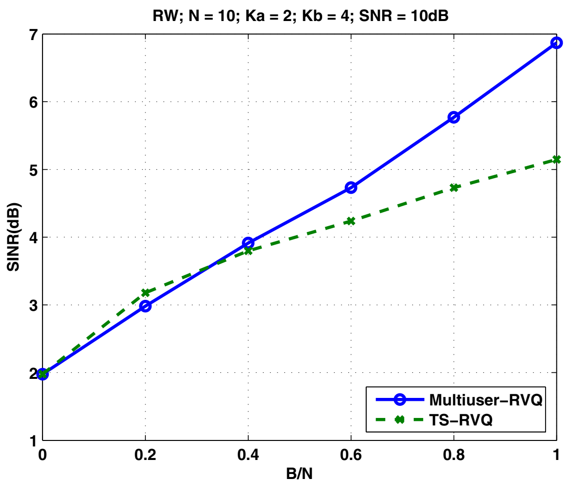

Fig. 3. An average SINR for both RVQ and TS-RVQ with normalized feedback bit is shown for a frequency-selective fading channel with path loss. The system parameters are $N = 10$, $K_a = 2$, $K_b = 4$, $L = 2$, $\alpha = 3$ and SNR = 10 dB.

V. Conclusions

We have proposed a tree structured RVQ scheme to quantize signatures for multiple users in DS-CDMA. TS-RVQ gives a lower average SINR for a given amount of feedback than RVQ does. However, complexity of RVQ increases exponentially with number of feedback bits while that of TS-RVQ increases linearly. For a given SINR, complexity of TS-RVQ can be an order of magnitude less than that of RVQ. Although the search for TS-RVQ is less complex, constructing a TS-RVQ codebook is much more computationally intensive than RVQ is . Fortunately, the codebook construction can be done offline. Here we only consider a reverse link where the receiver performs a centralized signature quantization for users. A quantization algorithm for a forward link, on the other hand, need to be distributed and will be our future work.

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