Antenna Subset Selection for MIMO Channels With Mutual Coupling

Dithanon Khrutmuang and Wiroonsak Santipach
Department of Electrical Engineering
Faculty of Engineering, Kasetsart University
Bangkok, Thailand 10900
Email: dithanon@hotmail.com and wiroonsak.s@ku.ac.th

Abstract—In this work, we propose antenna selection schemes for a point-to-point multiantenna channel. The schemes select subsets of either transmit and/or receive antennas to activate to increase ergodic capacity. The channel model considered includes spatial correlation and mutual coupling among adjacent antennas, which can have significant impact on the channel performance. Our proposed schemes are shown to perform close to the optimal exhaustive search with less computational complexity.

I. INTRODUCTION

A capacity of a multiple-input and multiple-output (MIMO) wireless channel has been shown to increase linearly with the minimum number of transmit and receive antennas [1], [2]. However, the linear capacity growth is applied only to a fading channel model with ideal scattering and no mutual coupling among transmitting or receiving antennas. If the scattering in the environment is not sufficiently rich, then there exists spatial correlation among channel gains between transmit and receive antenna pair. Mutual coupling is an electromagnetic phenomenon, which intrinsically exists in an antenna array, and its effect is detrimental to the capacity when antenna is placed sufficiently close to one another [3], [4]. These practical factors could affect the performance of a MIMO channel significantly and should be taken into account in any reasonable channel model.

To increase the performance of multiantenna link, we propose antenna selection schemes that incorporate mutual coupling consideration. Selecting subset of antennas has been shown to increase channel capacity. Furthermore, using only the selected set of antennas also reduces hardware complexity, e.g., a number of installed radio-frequency (RF) chains needed on either ends of communication link. Antenna selection has been long considered in the literature [5]–[10]. References [6]–[9] proposed fast suboptimal selection algorithms, which perform close to the exhaustive search over all possible antenna subsets. Assuming MIMO channel with spatial correlations between antennas, [10] proposed the scheme to select receive antennas based on correlations.

All of the work previously mentioned do not consider mutual coupling among antennas, which is shown to greatly affect capacity when antennas are placed very close to one

another [3], [4]. In this work, we propose suboptimal antenna selection schemes for MIMO channel model with both spatial correlation and mutual coupling. The objective of the proposed schemes is to maximize the capacity over potential antenna subsets whose mutual impedance is not to exceed a given threshold. From numerical examples shown, our schemes perform close to the exhaustive search, but require less computation time.

II. CHANNEL MODEL

We consider a point-to-point MIMO channel with N_t transmit and N_r receive antennas. For a discrete-time channel model, the $N_r \times 1$ received vector is given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{H}$ is an $N_r \times N_t$ channel matrix whose element $h_{i,j}$ is the channel gain between the jth transmit and the ith receive antennas, $\boldsymbol{x}$ is an $N_t \times 1$ transmitted vector with zero mean and covariance $\boldsymbol{I}$, which is the identity matrix. The transmitted signal is corrupted by additive white Gaussian noise (AWGN) vector $\boldsymbol{n}$, whose mean and covariance are zero and $\sigma_n^2 \boldsymbol{I}$, respectively.

Channel gain $h_{i,j}$ depends on channel model, antenna type, and antenna-array arrangement. Placing antennas close to one another induces a mutual coupling between them. Taking into account both effects of spatial correlation and mutual coupling, the channel matrix is given by

$$\boldsymbol{H} = \frac{1}{c_t c_r} \boldsymbol{C}_r \boldsymbol{R}^{\frac{1}{2}} \boldsymbol{H}_w \boldsymbol{S}^{\frac{1}{2}} \boldsymbol{C}_t \tag{2}$$

where, for an ideal Rayleigh fading channel, $\boldsymbol{H}_w$ is an $N_r \times N_t$ matrix of independent zero-mean unit-variance complex Gaussian random variables, $\boldsymbol{S}$ is the $N_t \times N_t$ transmit correlation matrix, $\boldsymbol{R}$ is the $N_r \times N_r$ receive correlation matrix, the $N_t \times N_t$ matrix $\boldsymbol{C}_t$ and $N_r \times N_r$ matrix $\boldsymbol{C}_r$ reflect mutual coupling effect at the transmit-antenna and receive-antenna arrays, respectively, and c_t and c_r are normalizing constants.

We assume that each antenna has a uniformly distributed angular spread. Thus, from Jakes' model [11], the (i, j) element of the transmit correlation matrix is given by

$$\boldsymbol{S}_{i,j} = J_0 \left(\frac{2\pi d_{i,j}}{\lambda} \right), \quad 1 \le i, j \le N_t \tag{3}$$

where $J_0(\cdot)$ is the zeroth-order Bessel function, and $d_{i,j}$ is the distance between the ith and jth transmit antennas, and

λ is the transmission wavelength. We assume that antennas are placed uniformly and the length of the antenna array is L. Hence,

$$d_{i,j} = \frac{L|i-j|}{N_t - 1}. \tag{4}$$

The receive correlation matrix $\boldsymbol{R}$ is similarly defined as the transmit correlation matrix $\boldsymbol{S}$ in (3).

According to [12], the two matrices reflecting mutual coupling effect at the transmit and receive arrays are given by

$$\boldsymbol{C}_t = \boldsymbol{Z}_t (\boldsymbol{Z}_t + \boldsymbol{Z}_s)^{-1} \tag{5}$$

$$\boldsymbol{C}_r = (\boldsymbol{Z}_r + \boldsymbol{Z}_l)^{-1} \tag{6}$$

where $\boldsymbol{Z}_t$ is an $N_t \times N_t$ transmit impedance matrix whose diagonal elements are self impedance and whose off-diagonal elements are mutual impedance, and $\boldsymbol{Z}_s$ denotes a diagonal matrix whose diagonal elements are the conjugate of the diagonal entries of $\boldsymbol{Z}_t$. For a dipole array, $\boldsymbol{Z}_t$ is well known from standard antenna theory (e.g., see [12, Ch. 8]). Similarly, $\boldsymbol{Z}_r$ is the $N_r \times N_r$ receive impedance matrix and $\boldsymbol{Z}_l$ is a diagonal matrix whose diagonal entries are the conjugate of the diagonal entries of $\boldsymbol{Z}_r$. Finally, the two normalizing constants are defined as follows

$$c_t = \frac{(\boldsymbol{Z}_t)_{11}}{(\boldsymbol{Z}_t)_{11} + (\boldsymbol{Z}_t)_{11}^*} \tag{7}$$

$$c_r = \frac{(\boldsymbol{Z}_r)_{11}}{(\boldsymbol{Z}_r)_{11} + (\boldsymbol{Z}_r)_{11}^*} \tag{8}$$

where $(\cdot)^*$ denotes complex conjugate.

Assuming that the channel matrix is known perfectly at the transmitter and receiver, we would like to maximize the performance of the channel by selectively activating subsets of transmit and/or receive antennas. For a transmit-antenna selection, we let $\boldsymbol{F}_t$ denote an $N_t \times N_t$ diagonal matrix whose ith diagonal entry is 1 if the ith transmit antenna is chosen or 0, otherwise. Similarly, we let $\boldsymbol{F}_r$ denote the $N_r \times N_r$ receive-antenna selection matrix. Let L_t be the number of selected or activated transmit antennas, and L_r be the number of selected receive antennas. Thus, after the transmit and receive selections, the effective channel matrix is given by $\boldsymbol{F}_r \boldsymbol{H} \boldsymbol{F}_t$ and the associated ergodic capacity is given by

$$C = E\left[\log \det(\boldsymbol{I} + \frac{\rho}{L_t} \boldsymbol{F}_r \boldsymbol{H} \boldsymbol{F}_t \boldsymbol{H}^\dagger \boldsymbol{F}_r)\right] \tag{9}$$

where the expectation is over channel distribution and $\boldsymbol{F}_t \boldsymbol{F}_t^\dagger = \boldsymbol{F}_t$. We note that antennas that are not selected still induce coupling with adjacent antennas. However, we assume that this mutual coupling from passive antennas is small and can be discarded. Thus, (9) is also a capacity of the link equipped with selected antennas only and all other antennas removed.

In this work, we propose algorithms to determine $\boldsymbol{F}_t$ and/or $\boldsymbol{F}_r$ that maximize channel capacity for the channel model with mutual coupling.

III. SELECTION ALGORITHMS

In this section, we describe the proposed algorithms to select a set of transmit antennas, a set of receive antennas, or a set of both transmit and receive antennas. The optimal selection scheme that maximizes the channel capacity employs exhaustive search over all possible subsets of antennas. However, search complexity can be tremendous when the number of antenna is large. Here we propose the selection schemes that take into account mutual coupling, reduce the search complexity, and perform close to the optimum.

A. Selection At Either Transmit or Receive Antenna Arrays

We start with selection for a single antenna array at either end of a communication link. This selection algorithm is based on that proposed by [9], [13] in which a single antenna is chosen at each step sequentially. To select L_t transmit antennas, we assume that all receive antennas are active ($L_r = N_r$ and thus, $\boldsymbol{F}_r = \boldsymbol{I}$). Let $\boldsymbol{e}_j$ be an $N_t \times 1$ vector whose jth entry is one and all other entries are zero and $\boldsymbol{e}_j$ also indicates selecting the jth antenna. For example, $\boldsymbol{e}_3 = [0\ 0\ 1\ 0\ \cdots 0]^T$. We also let Ω_s and Ω_n be the sets of $\boldsymbol{e}_j$'s that correspond to selected antennas and the remaining antennas, respectively.

The proposed algorithm starts with empty Ω_s and a full $\Omega_n = \{\boldsymbol{e}_1, \dots, \boldsymbol{e}_{N_t}\}$. Given channel matrix $\boldsymbol{H}$, we select the single transmit antenna that maximizes an instantaneous capacity. If jth antenna is selected, we move the corresponding $\boldsymbol{e}_j$ from Ω_n to Ω_s. To select subsequent antennas, we determine first if a ratio between mutual impedance between a potential antenna in Ω_n and all selected antennas in Ω_s and the self impedance exceeds a threshold ϵ, where $0 < \epsilon < 1$. If the ratio exceeds ϵ, we will not consider that antenna as a candidate. From a set of candidate antennas denoted by Ω_c, we again find the antenna that maximizes the instantaneous capacity and add it to Ω_s. If Ω_c is empty, we set Ω_c to equal Ω_n. We iterate until L_t antennas are chosen. Detailed steps are shown in Algorithm 1. We note that ϵ limits the number of candidate antennas in Ω_c. If ϵ is set too low, Ω_c may contain too few antennas. If ϵ is set too high, Ω_c will be similar to Ω_u. In this work, we obtain ϵ from numerical experiments.

To select receive antennas, similar steps apply. We set $L_t = N_t$ and hence, $\boldsymbol{F}_t = \boldsymbol{I}$. Similar to the transmit selection in Algorithm 1, receive antennas are selected sequentially based on instantaneous capacity and mutual impedance.

B. Selection At Both Transmit and Receive Antenna Arrays

We would also like to select *both* transmit and receive antennas that maximize the capacity. Employing exhaustive search may not possible since the number of possible sets of transmit and receive antennas, which equals $\binom{N_t}{L_t}\binom{N_r}{L_r}$, could be forbiddingly high. To lessen search complexity, we can apply Algorithm 1 to select first the transmit antennas and then the receive antennas or vice versa. In other words, Algorithm 1 is used twice in sequence and the number of possible capacity computations will be at most $\binom{N_t}{L_t} + \binom{N_r}{L_r}$. However, the performance consequently suffers as later shown by numerical examples in Section IV.

To improve upon the performance of Algorithm 1, we propose Algorithm 2, which is based on the exhaustive search. However, the number of the possible subsets of antennas is reduced by a constraint on mutual impedance. For either transmit or receive sides, we only consider the set of antennas

Algorithm 1 Selection of transmit antennas

1: $\Omega_s = \{\}$ and $\Omega_n = \{e_1, \ldots, e_{N_t}\}$.
2: $L = 0$
3: **while** $L < L_t$ **do**
4:    **if** $L = 0$ **then**
5:       Find
$$e_s = \arg \max_{e_j \in \Omega_n} \log \det(I + \frac{\rho}{L_t} F_r H e_j e_j^\dagger H^\dagger F_r).$$
6:       $\Omega_s \leftarrow \Omega_s \cup \{e_s\}$
7:       $\Omega_n \leftarrow \Omega_n \backslash \{e_s\}$
8:       $L \leftarrow L + 1$
9:    **else**
10:       $\Omega_c = \{\}$
11:       **for all** $e_t \in \Omega_s$ **do**
12:          $\Omega_c \leftarrow \Omega_c \cup \left\{ e_w \in \Omega_n \left| \frac{|e_t^\dagger Z_t e_w|}{|Z_{t;1,1}|} \leq \epsilon \right. \right\}$
13:       **end for**
14:       **if** $\Omega_c = \{\}$ **then**
15:          $\Omega_c \leftarrow \Omega_n$
16:       **end if**
17:       Find
$$e_s = \arg \max_{e_j \in \Omega_c} \log \det(I + \frac{\rho}{L_t} F_r H (\sum_{e_i \in \Omega_s} e_i e_i^\dagger + e_j e_j^\dagger) H^\dagger F_r).$$
18:       $\Omega_s \leftarrow \Omega_s \cup \{e_s\}$
19:       $\Omega_n \leftarrow \Omega_n \backslash \{e_s\}$
20:       $L \leftarrow L + 1$
21:    **end if**
22: **end while**
23: **return** Ω_s

Algorithm 2 Selection of transmit antennas and receive antennas

1: Start with $\mathcal{F}_t$ and $\mathcal{F}_r$.
2: Find
$$\bar{\mathcal{F}}_t = \{F_t \in \mathcal{F}_t \mid \text{all nonzero off-diagonal entries}$$
$$\text{of } F_t Z_t F_t < \epsilon Z_{t;1,1}\}.$$
3: Find
$$\bar{\mathcal{F}}_r = \{F_r \in \mathcal{F}_r \mid \text{all nonzero off-diagonal entries}$$
$$\text{of } F_r Z_r F_r < \epsilon Z_{r;1,1}\}.$$
4: Find
$$F_t^*, F_r^* = \arg \max_{\substack{F_t \in \bar{\mathcal{F}}_t \\ F_r \in \bar{\mathcal{F}}_r}} \log \det(I + \frac{\rho}{L_t} F_r H F_t H^\dagger F_r).$$
5: **return** F_t^* and F_r^*

---

whose ratio between mutual impedance and self impedance does not exceed the threshold ϵ.

Let $\mathcal{F}_t = \{F_t^{(i)}\}$, where $1 \leq i \leq \binom{N_t}{L_t}$, denote the set of all possible transmit-antenna selection matrices and $\mathcal{F}_r = \{F_r^{(j)}\}$, where $1 \leq j \leq \binom{N_r}{L_r}$, denote the set of all possible receive-antenna selection matrices. We note that nonzero off-diagonal entries of $F_t^{(i)} Z_t F_t^{(i)}$ are mutual impedance of the transmit antennas associated with $F_t^{(i)}$. Similarly, nonzero off-diagonal entries of $F_r^{(j)} Z_r F_r^{(j)}$ are mutual impedance of the receive antennas associated with $F_r^{(j)}$. Steps of this proposed algorithm is shown in Algorithm 2.

IV. NUMERICAL RESULTS

In all simulation results shown, we assume dipole antenna whose radius is 0.001λ and whose length is 0.25λ. All antennas in the array are uniformly placed side by side. In Fig 1, an ergodic capacity of transmit selection scheme in Algorithm 1 is shown with N_t for $\rho = 0$ dB, $L_t = 2$, $N_r = L_r = 3$, and threshold $\epsilon = 0.8$. Lengths of both arrays and are set to $L = \lambda$. When N_t is large, transmit antennas are placed very close together and hence, induce a large mutual coupling. From the figure, our proposed scheme, exhaustive search, and the scheme proposed by Gorokhov [8] performs

similarly. Similar to [13], we modify the Gorokhov scheme by starting with an empty set of selected antennas and adding one antenna at each step to the set. However, based on simulation results not shown here, our scheme is the least complex among the three schemes[1] We also plot the capacity of selecting all transmit antennas and see a significant performance loss.
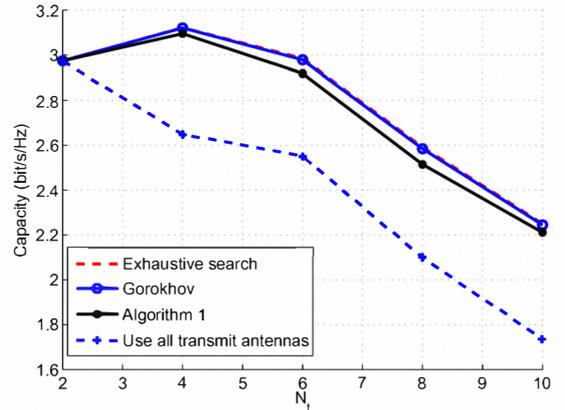
Fig. 1. Capacity with transmit selection scheme in Algorithm 1 is shown with N_t for $\rho = 0$ dB, $L_t = 2$, $N_r = L_r = 3$, $L = \lambda$, and $\epsilon = 0.8$.

With $N_t = N_r = 10$, we would like to find the optimal number of selected transmit and receive antennas. Fig. 2 shows the capacity versus L_t or L_r. For each plot, algorithm 1 was applied to either transmit or receive arrays while all antennas in the other array are active. With transmit selection, the optimal $L_t = 3$ when $\rho = 5$ dB and lengths of both arrays equal 0.5λ. Comparing to using all transmit antennas, we can gain almost 20% from activating only subset of antennas. For receive-antenna selection, using all receive antennas gives the capacity maximum. This is due to the fact that more energy is captured by a larger number of receive antennas. However, more RF chains also need to be installed with a larger cost.

---

[1]The associated numerical result is not shown here due to limited space.
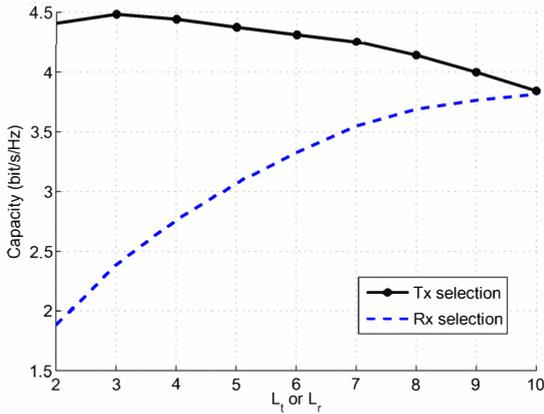
Fig. 2. Capacity with either transmit or receive selection scheme is shown with L_t or L_r for $\rho = 5$ dB, $N_t = N_r = 10$, $L = 0.5\lambda$, and $\epsilon = 0.8$.

Fig. 4. Computation time of the schemes whose achievable rates are shown in Fig. 3 is plotted with the number of transmit and receive antennas.

In Fig. 3 and 4, both transmit *and* receive antennas are selected with $\rho = 0$ dB, $L_t = L_r = 3$, $L = 0.5\lambda$, and $N_t = N_r$. We note that thresholds for Algorithms 1 and 2 are 0.8 and 0.3, respectively. We apply Algorithm 2 to find the set of selected transmit and receive antennas, which is then compared to the exhaustive search and also Algorithm 1. All three schemes give similar rate performance with the maximum attained when $N_t = N_r = 5$. Complexity of the three schemes is measured by the computation time and is shown in Fig. 4. We note that computation time of exhaustive search increases exponentially with the number of available antennas while those of our proposed schemes are less, especially computation time of Algorithm 1.

Fig. 3. Capacity with selection of both transmit and receive antennas is shown with N_t or N_r for $\rho = 0$ dB, $L_t = L_r = 3$, $L = 0.5\lambda$, $\epsilon_1 = 0.8$, and $\epsilon_2 = 0.3$.

V. CONCLUSIONS

We proposed antenna selection algorithms for a single-user MIMO channel, which takes into account mutual coupling effect. The proposed schemes shown to perform close to an exhaustive search with much less complexity. Search complexity is a major issue since the selection of antennas happens in

real time and cannot be done offline. Our search complexity depends heavily on the threshold on mutual impedance, which is currently obtained from numerical simulations. Also, performance of the proposed schemes is shown via numerical results. Thus, our future work includes analysis of the performance, which should give us more insights, as well as investigation on the optimal threshold for a given setup.

REFERENCES

[1] İ. E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. on Telecommun.*, vol. 10, pp. 585–595, Nov. 1999.

[2] T. Marzetta and B. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inf. Theory*, vol. 45, no. 1, pp. 139 –157, Jan. 1999.

[3] S. Shen, M. McKay, and R. Murch, "MIMO systems with mutual coupling: How many antennas to pack into fixed-length arrays?" in *Int. Symp. on Inf. Theory and its Applications (ISITA)*, Oct. 2010, pp. 531 –536.

[4] J. Wallace and M. Jensen, "Mutual coupling in MIMO wireless systems: A rigorous network theory analysis," *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1317 – 1325, Jul. 2004.

[5] M. Win and J. Winters, "Analysis of hybrid selection/maximal-ratio combining in Rayleigh fading," *IEEE Trans. Commun.*, vol. 47, no. 12, pp. 1773 –1776, Dec. 1999.

[6] D. Gore, R. Nabar, and A. Paulraj, "Selecting an optimal set of transmit antennas for a low rank matrix channel," in *IEEE Int. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 5, 2000, pp. 2785 –2788 vol.5.

[7] S. Sandhu, R. Nabar, D. Gore, and A. Paulraj, "Near-optimal selection of transmit antennas for a MIMO channel based on shannon capacity," in *34th Asilomar Conf. on Signals, Systems and Computers*, vol. 1, 2000, pp. 567 –571 vol.1.

[8] A. Gorokhov, D. Gore, and A. Paulraj, "Receive antenna selection for MIMO spatial multiplexing: Theory and algorithms," *IEEE Trans. Signal Process.*, vol. 51, no. 11, pp. 2796 – 2807, Nov. 2003.

[9] A. Gorokhov, "Antenna selection algorithms for MEA transmission systems," in *IEEE Int. Conf. on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 3, May 2002, pp. III–2857 –III–2860.

[10] L. Dai, S. Sfar, and K. Letaief, "Optimal antenna selection based on capacity maximization for MIMO systems in correlated channels," *IEEE Trans. Commun.*, vol. 54, no. 3, pp. 563 – 573, Mar. 2006.

[11] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1975.

[12] C. A. Balanis, *Antenna Theory: Analysis and Design*. New York: Wiley, 1997.

[13] M. Gharavi-Alkhansari and A. B. Gershman, "Fast antenna subset selection in MIMO systems," *IEEE Trans. Signal Process.*, vol. 52, no. 2, pp. 339–346, Feb. 2004.