On Optimizing Transmit Antenna Placement for Downlink Distributed Antenna Systems With Zero-Forcing Beamforming

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***Abstract*—We consider a downlink distributed antenna system (DAS) in which transmit antennas are placed in a circular layout and each mobile user is equipped with a single receive antenna. Assuming zero-forcing beamforming transmission, a base station randomly selects users to which it transmits data. We derive the upper bound on a sum of all users' achievable rates and use the bound to estimate the optimal radius of the antenna layout that maximizes the sum rate. Numerical examples show that the optimal radius depends on the number of transmit antennas, signal-to-noise ratio, and variance of user density. The sum rate of DAS with optimized placement of transmit antennas can be significantly higher than that of a co-located antenna system (CAS).**

***Index Terms*—Distributed antenna system (DAS), circular antenna layout, downlink, zero-forcing beamforming, nonuniform user density, sum-rate analysis.**

I. Introduction

Multiple-antenna communication can increase spectral efficiency and diversity. In a traditional downlink transmission, all transmit antennas are located at the base station, which is at the cell center. In recent years, a distributed antenna system (DAS) in which transmit antenna ports are distributed throughout the cell, has gained interest due to its improved coverage and capacity over a co-located antenna system (CAS) [1], [2].

In this work, we assume that the base station employs a zero-forcing beamforming transmission in which a transmit beamforming vector for a user is selected such that there is no interference affecting other users. Users served at any given time are randomly selected by the base station. Zero-forcing beamforming is much simpler than dirty-paper coding, and approaches the optimum as the number of users tends to infinity [3]. To maximize a sum ergodic rate in a multiple-input multiple-output (MIMO) downlink, a user selection scheme was proposed in [4] while the number of users served was optimized in [5], assuming a large number of transmit antennas. References [3]–[5] assumed co-located transmit antennas.

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For DAS, placement of distributed antenna (DA) ports in a cell can significantly affect the system performance [6]–[10]. In [6], single-antenna DA ports in either a single-cell or a two-cell model were assumed and location of DA ports was determined by maximizing the lower bound on received signal-to-noise ratio (SNR). The results shown in [6] were extended to DAS with multiple-antenna DA ports by [7]. Both work [6], [7] assumed uniform user distribution in the cell. In [8], a circular antenna layout with nonuniform user distribution was considered and optimized. In [10], the optimal layout radius was analyzed with a large system limit in which the number of DA ports and users tend to infinity with a fixed ratio. However, the system in [8], [10] was assumed to serve one user at a time. In [9], placement of DA ports was investigated for uplink DAS whose performance was measured by cell averaged symbol error rate.

In this work, we are interested in optimizing placement of single-antenna DA ports of a downlink single-cell DAS. All DA ports are placed in a circular layout whose center is at the cell center similar to models considered by [8]–[10]. We assume a zero-forcing beamforming transmission in which *multiple* users can be served by the base station with no interference. The number of users served at any given time is less than or equal to the number of DA ports. We analyze the upper bound of the sum achievable rate of all users, which depends on the number of DA ports, number of users, user distribution, and radius of the antenna layout. We show that the radius of the antenna layout that maximizes the analytical upper bound on the sum rate is a good approximation of the optimal radius that maximizes the sum rate. Our results apply to any user distribution with uniform angular distribution. Numerical example shows that DAS with the optimized DA port placement and a large number of DA ports can outperform CAS by as much as 25%.

II. System Model

We consider a single-cell downlink channel with a base station consisting of N distributed ports with single transmit antenna each and $K \leq N$ mobile stations or users each equipped with a single antenna. These DA ports are connected to a central processor via optical fiber. We assume the cell is circular with radius R and all DA ports are uniformly placed on the circular layout with radius $r \leq R$. There is no antenna port at the center of the cell. Fig. 1 illustrates the antenna layout with 6 DA ports shown with gray circles. This assumed antenna configuration has been shown to maximize a sum achievable rate with a moderate number of transmit antennas and uniform user distribution [6].

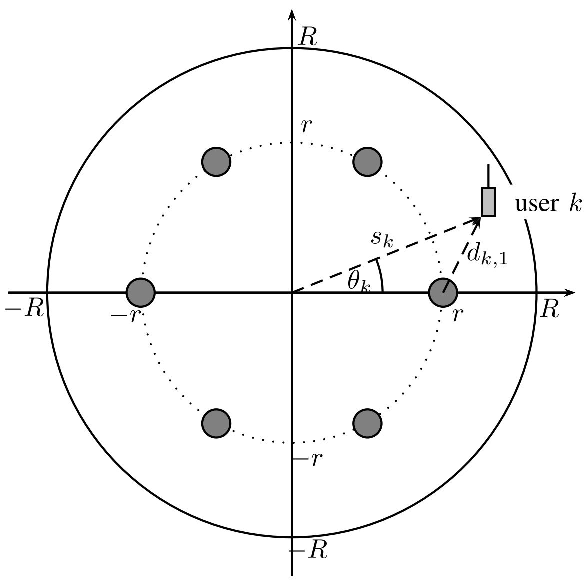

Fig. 1. A circular antenna layout with $N = 6$ is shown.

For a discrete-time model, a transmitted signal is denoted by the $N \times 1$ transmitted vector

$$\boldsymbol{x} = \sum_{k=1}^{K} \boldsymbol{w}_k b_k \tag{1}$$

where $\boldsymbol{w}_k$ is the $N \times 1$ transmit beamforming vector for user k and b_k is a transmitted symbol for user k. For user k, the received vector is given by

$$y_k = \boldsymbol{h}_k^* \boldsymbol{x} + n_k \tag{2}$$

where the $1 \times N$ channel vector for user k

$$\boldsymbol{h}_k = \left[g_{k,1} d_{k,1}^{-\frac{\alpha}{2}} \quad g_{k,2} d_{k,2}^{-\frac{\alpha}{2}} \quad \cdots \quad g_{k,N} d_{k,N}^{-\frac{\alpha}{2}} \right], \tag{3}$$

n_k is an additive white Gaussian noise with zero mean and variance σ_n^2, and $(\cdot)^*$ denotes complex conjugate. Channel vector in (3) reflects both large and small scale fading where $g_{k,n}$ denotes a fading coefficient between user k and DA port n, $d_{k,n}$ denotes the distance between user k and DA port n, and α denotes a path-loss exponent. We note that the fading channel for each pair of user and transmit antenna is assumed to be flat. Assuming ideal scattering and uniform channel gain, $g_{k,n}$'s are independent complex Gaussian with zero mean and unit variance. $d_{k,n}^{-\frac{\alpha}{2}}$ accounts for signal degradation with α ranging between 2 and 4, depending on the terrain and propagation environment.

Substituting (1) into (2) gives

$$y_k = \boldsymbol{h}_k^* \boldsymbol{w}_k b_k + \boldsymbol{h}_k^* (\sum_{\substack{i=1 \\ i \neq k}}^{K} \boldsymbol{w}_i b_i) + n_k \tag{4}$$

where the second term on the right-hand side is the interference caused by other users. Thus, the corresponding achievable rate for user k is given by

$$R_k = \log_2 \left(1 + \frac{P_k |\boldsymbol{h}_k^* \boldsymbol{w}_k|^2}{\sigma_n^2 + \sum_{\substack{i=1 \\ i \neq k}}^{K} P_i |\boldsymbol{h}_k^* \boldsymbol{w}_i|^2} \right) \tag{5}$$

where the transmitted power for user k is $P_k = E|b_k|^2$.

With zero-forcing beamforming transmission, user k 's transmit beamforming vector $\boldsymbol{w}_k$ is selected such that there is no interference from other users as follows

$$\boldsymbol{h}_i^* \boldsymbol{w}_k = 0 \quad \text{for} \quad \forall i \neq k. \tag{6}$$

Let $\boldsymbol{W} = [\boldsymbol{w}_1 \quad \boldsymbol{w}_2 \quad \cdots \quad \boldsymbol{w}_K]$ denote an $N \times K$ matrix whose columns are transmit beamforming vectors. The zero-forcing solution is given by [11]

$$\boldsymbol{W} = \boldsymbol{H}^\dagger (\boldsymbol{H}\boldsymbol{H}^\dagger)^{-1} \tag{7}$$

where the $K \times N$ channel matrix

$$\boldsymbol{H} = \left[\boldsymbol{h}_1^T \quad \boldsymbol{h}_2^T \quad \cdots \quad \boldsymbol{h}_N^T \right]^T \tag{8}$$

and $(\cdot)^\dagger$ and $(\cdot)^T$ denote Hermitian transpose and transpose, respectively. The associated achievable rate for user k is given by

$$R_k = \log_2 \left(1 + \frac{P_k \gamma_k}{\sigma_n^2} \right) \tag{9}$$

where the effective channel gain for user k is the inverse of the (k,k) entry of $(\boldsymbol{H}\boldsymbol{H}^\dagger)^{-1}$ as follows [12]

$$\gamma_k = \frac{1}{[(\boldsymbol{H}\boldsymbol{H}^\dagger)^{-1}]_{k,k}} \tag{10}$$

and the expected sum rate over all channel realization is given by

$$\mathcal{R} = \sum_{k=1}^{K} E[R_k]. \tag{11}$$

The sum rate in (11) is a function of r, which is a distance between all DA ports to the cell center. We would like to find the optimal r that maximizes the sum rate as follows:

$$r_{\text{opt}} = \arg \max_{0<r\leq R} \mathcal{R}. \tag{12}$$

The optimization problem in (12) can not be solved analytically since a closed-form expression for the expected sum rate $\mathcal{R}$ is not tractable.

III. Optimizing Antenna Placement

Instead of directly analyze $\mathcal{R}$, we derive its upper bound. First, for each user k, we define the $1 \times N$ channel vector $\bar{\boldsymbol{h}}_k$ with all distances between user k and all DA ports replaced with the minimum distance as follows

$$\bar{\boldsymbol{h}}_k \triangleq (d_k^{\min})^{-\frac{\alpha}{2}} \begin{bmatrix} g_{k,1} & g_{k,2} & \cdots & g_{k,N} \end{bmatrix} \tag{13}$$

where

$$d_k^{\min} = \min_{1 \le n \le N} d_{k,n}. \tag{14}$$

Thus, the expected strength of the channel with the minimum distance is larger or equal to that of the actual channel

$$E|\boldsymbol{h}_k|^2 \le E|\bar{\boldsymbol{h}}_k|^2. \tag{15}$$

We define the effective channel gain with $d_k^{\min}$ as follows

$$\bar{\gamma}_k \triangleq \frac{1}{[(\bar{\boldsymbol{H}}\bar{\boldsymbol{H}}^{\dagger})^{-1}]_{k,k}} \tag{16}$$

where $\bar{\boldsymbol{H}} = \begin{bmatrix} \bar{\boldsymbol{h}}_1^T & \bar{\boldsymbol{h}}_2^T & \cdots & \bar{\boldsymbol{h}}_N^T \end{bmatrix}^T$. Because of (15), replacing all distances between user and antenna ports with the minimum distance gives the upper bound on the effective channel gain as follows

$$\gamma_k \le \bar{\gamma}_k. \tag{17}$$

In other words, the upper bound $\bar{\gamma}_k$ is obtained from the system in which all transmit antennas are co-located with distance $d_k^{\min}$ away from user k.

With the result in [12], we can derive the probability density function (pdf) for $\bar{\gamma}_k$ given by

$$f_{\bar{\gamma}_k}(x) = \frac{(d_k^{\min})^{\alpha}}{(N-K)!} \left((d_k^{\min})^{\alpha} x\right)^{N-K} \mathrm{e}^{-(d_k^{\min})^{\alpha} x}. \tag{18}$$

Hence, the upper bound on the achievable rate for user k is given by

$$R_k \le \bar{R}_k \triangleq \log_2 \left(1 + \frac{P_k \bar{\gamma}_k}{\sigma_n^2}\right). \tag{19}$$

Conditioned on the minimum distance $d_k^{\min}$, the expected upper bound for user k can be evaluated as follows

$$E_{\boldsymbol{H}}\left[\bar{R}_k | d_k^{\min}\right] = \int_0^{\infty} \log_2 \left(1 + \frac{P_k x}{\sigma_n^2}\right) f_{\bar{\gamma}_k}(x)\,\mathrm{d}x. \tag{20}$$

where the expectation is over channel realization.

We assume that all K users are randomly selected by the base station. Hence, $d_k^{\min}$ is a random variable. Next we will average $E\left[\bar{R}_k | d_k^{\min}\right]$ over $d_k^{\min}$. Without loss of generality, we assume that the DA port that is closest to the user is located at the point $(r, 0)$ in a Cartesian coordinate in which the cell center is located at the origin (see Fig. 1). The minimum distance can be expressed as [10]

$$d_k^{\min} = \sqrt{s_k^2 + r^2 - 2 s_k r \cos(\theta_k)} \tag{21}$$

where s_k denotes the distance from user k to the cell center and θ_k is the polar angle between the antenna located at $(r, 0)$ and the user. Fig. 1 shows s_k, θ_k, and $d_{k,1}$, which equals $d_k^{\min}$ for the shown location of user k. In general, s_k and θ_k are random variables whose pdf's depend on how users are distributed in the cell. Here we assume that users are located uniformly in all angles. Thus, the pdf for θ_k is given by

$$f_{\theta_k}(\theta) = \frac{1}{2\pi}, \text{ for } -\pi \le \theta \le \pi. \tag{22}$$

Combining (20), (21), and (22), we obtain the expected upper bound as follows

$$E[\bar{R}_k] = \int_0^R \left(\int_{-\pi}^{\pi} E_{\boldsymbol{H}}\left[\bar{R}_k | d_k^{\min}\right] \frac{1}{2\pi}\,\mathrm{d}\theta \right) f_{s_k}(s)\,\mathrm{d}s \tag{23}$$

where $f_{s_k}(\cdot)$ denotes the pdf for s_k. The two popular types of user density are uniform and Gaussian or normal. For uniform density,

$$f_{s_k}(s) = \frac{2s}{R^2}, \text{ for } 0 \le s \le R, \tag{24}$$

and for Gaussian or normal density,

$$f_{s_k}(s) = \frac{s}{\sigma^2} \mathrm{e}^{-\frac{s^2}{2\sigma^2}}, \text{ for } 0 \le s \le R \tag{25}$$

where σ^2 denotes the variance of Gaussian density. Substituting either (24) or (25) or other pdf's for s_k into (23), the upper bound in (23) can be evaluated by any conventional numerical method.

We define the upper bound for the sum rate as follows

$$\mathcal{R} \le \bar{\mathcal{R}} = \sum_{k=1}^{K} E[\bar{R}_k]. \tag{26}$$

With the assumptions that all user signals are transmitted with equal power and are propagated through independent identically distributed channels, and all users are randomly selected, the upper bound on the rate is then the same for all users. We can find the antenna layout radius that maximizes the upper bound as follows

$$\bar{r}_{\text{opt}} = \arg \max_{0 \le r \le R} \bar{\mathcal{R}} = \arg \max_{0 \le r \le R} E[\bar{R}_1]. \tag{27}$$

We note that $\bar{r}_{\text{opt}}$ found from (27) may not be the same as the optimal radius r_{opt}. However, from results of numerical simulations that we have seen, $\bar{r}_{\text{opt}}$ is usually close or equal to r_{opt}.

IV. Numerical Results

In this section, we compare simulation results obtained from Monte Carlo simulations with analytical results derived in Section III. We assume that the cell radius R is 100 meters, the path-loss exponent α is 3, and transmitted power for all users is equal.

In Fig. 2, we assume DA ports are installed at radius $r = 60$ meters and user density is uniform. Shown are the sum rate for two DAS's with $N = K = 6$ and $N = 20, K = 15$. As expected, the sum rate increases with SNR. We verify the analytical bound derived in Section III with the simulation of CAS with the same set of minimum distances as DAS, and see that the analytical results match with the simulation ones. However, with the given r, the analytical upper bounds are loose and can be twice as large as the actual sum rate. We

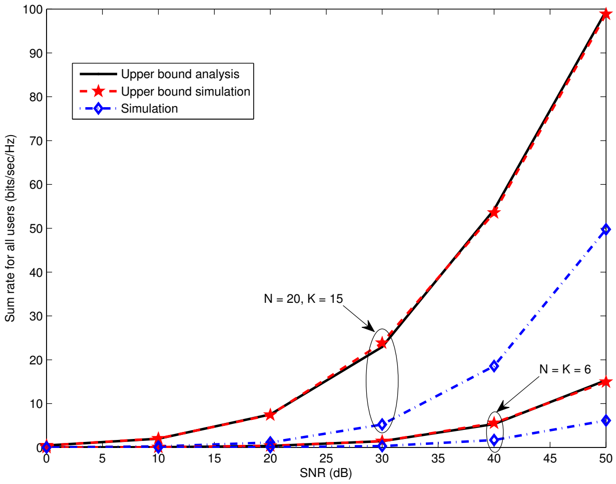

Fig. 2. Analytical bounds and actual sum rates are shown with different SNR for $r/R = 0.6$, and for two system sizes $N = 20, K = 15$ and $N = K = 6$.

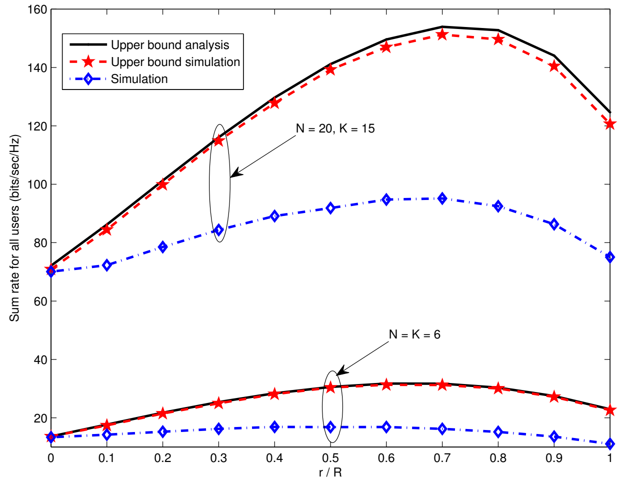

Fig. 3. Sum rates are plotted with different r/R for SNR at 60 dB and two system sizes: $N = 20, K = 15$ and $N = K = 6$.

expect the bound to be tighter when r/R is very close to zero.

In Fig. 3, sum rates are plotted with different radii r for DAS's with $N = K = 6$ and $N = 20, K = 15$ at SNR 60 dB. We observe the peak sum rate occurs at $r/R = 0.7$ for both the upper bound and the actual sum rate for DAS with $N = 20$ and $K = 15$. Similarly, the optimal radius $r/R = 0.6$ for both the upper bound and the actual sum rate for DAS with $N = K = 6$. Although the analytical upper bound is loose and is only tight at $r = 0$, it gives the radius that closely matches with the optimal radius that maximizes the sum rate. From the figure, performance at $r = 0$ is that of CAS. We note that for the smaller system, i.e., $N = K = 6$, the sum rate of CAS is comparable to that of DAS. However, for the larger system, DAS with the optimal radius can outperform CAS by as much as 25%.

Fig. 4 shows simulated sum rates with different SNR's for DAS with $N = 3$ and $K = 3$. Dash-dot lines denote sum rates with the optimal radii r_{opt} while solid lines denote sum rates with radii $\bar{r}_{\text{opt}}$ obtained from (27). We note that $\bar{r}_{\text{opt}}$, which is obtained from the upper bound analysis, performs close to the optimum. The gap between the two is minimal for low SNR and small for large SNR. Results for both uniform and normal user densities are shown. We see that normal user density results in larger sum rate than uniform density. In other words, sum rate is larger when users are more concentrated around the cell center. This is to be expected due to smaller degradation of signal power.

In Fig. 5, we assume users are placed randomly with uniform density in the cell and the number of transmit antennas is equal to that of users served ($N = K$). The optimal radius for the antenna placement normalized with the cell radius $\frac{r_{\text{opt}}}{R}$ is shown to increase with the number of transmit antennas and SNR. For low to moderate SNR, r_{opt} increases faster with

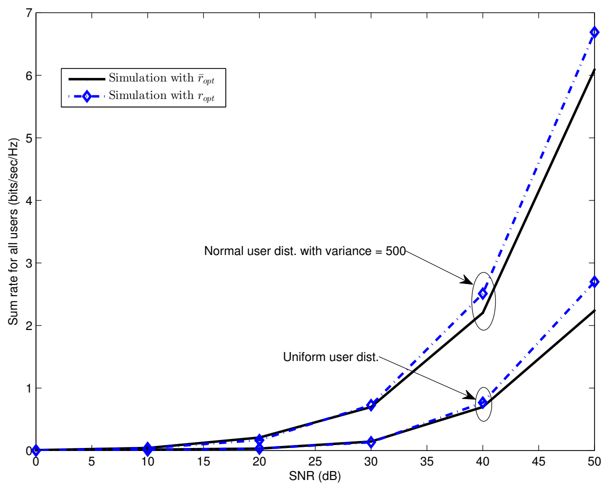

Fig. 4. Sum rates with DA ports placed at radius either r_{opt} or $\bar{r}_{\text{opt}}$ are shown with SNR for two types of user density.

the number of transmit antennas N. This is in contrast to a high SNR regime in which r_{opt} slowly increases with N. The results apply for a single-cell system or cellular system with a large frequency reuse factor. For the system with small frequency reuse factor, our results may not be accurate due to a substantial presence of inter-cell interference. Fig. 5 also compares r_{opt} with $\bar{r}_{\text{opt}}$. We see that generally $\bar{r}_{\text{opt}}$ can predict the optimal radius quite well.

The optimal radius is plotted with variance of Gaussian user density σ^2 in Fig. 6 for DAS with $N = K = 6$ and SNR at either 10 or 60 dB. We note that the radius that maximizes sum rate increases with σ^2. It is not difficult to see that as more users move away from the cell center (larger σ^2), the antenna ports should be placed further away from the cell center as

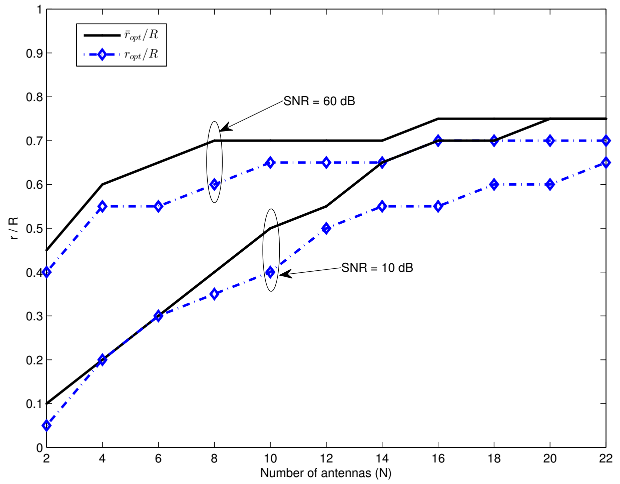

Fig. 5. The optimal radius for antenna placement r_{opt} is compared with the suboptimal $\bar{r}_{\text{opt}}$ obtained from the upper bound analysis for DAS with $N = K$ and SNR at either 10 or 60 dB.

well. Again we remark that from the figure, $\bar{r}_{\text{opt}}$ is close to r_{opt}.

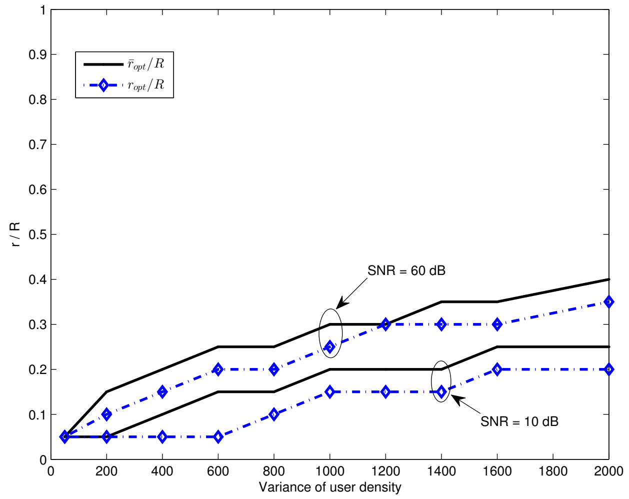

Fig. 6. Assuming Gaussian user density with variance σ^2, the optimal radius for antenna placement r_{opt} is compared with the suboptimal $\bar{r}_{\text{opt}}$ obtained from the upper bound analysis for different values of σ^2, $N = K = 6$, and SNR at either 10 or 60 dB.

V. Conclusions

We derived the upper bound on a sum rate for downlink DAS with circular antenna layout. The radius of the antenna layout that maximizes the analytical bound was shown to predict the optimal radius very well. We found that the optimal radius increases with the number of transmit antennas, SNR, and variance of user density. The numerical example showed that operating at the optimal radius in DAS can outperform CAS by as much as 25%. Thus, the analytical results derived in this work should provide a good practical guideline for deploying DAS. Although we only considered a single-cell channel in this work, the contribution should apply to any multi-cell model with large frequency-reuse factor as well. For a multi-cell system with small frequency-reuse factor or with dominant inter-cell interference, the optimal antenna layout remains a open problem. Here we considered only Rayleigh fading model. Other practical fading models can be considered in the future work.

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