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Chunk-Based Subcarrier Assignment With Power Allocation and Rate Constraints for Downlink OFDMA

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***Abstract*—We consider a downlink OFDMA in which users may have different rate requirements and a base station would like to maximize user rate over subcarrier assignment (SA) and power allocation (PA). To attain that objective, we propose suboptimal chunk-based SA and PA algorithms that are less complex and perform better than existing suboptimal schemes. Numerical results show that in certain example the proposed SA and PA can provide user rate up to 98.6% of the optimum rate obtained from a joint SA and PA problem.**

I. Introduction

Demand for data access is growing at a tremendous rate. However, resource such as spectrum and power is limited. Thus, optimizing resource allocation is an important problem to consider. An approach called rate adaptive (RA) allocation is commonly used to dynamically allocate the resource. The objective of RA is to maximize a total throughput while satisfying a total transmit power constraint [1]. In Rhee and Cioffi's RA work [2], which assumes orthogonal frequency-division multiple access (OFDMA), rate fairness is considered as another constraint and is achieved by maximizing the minimum user throughput. However, this approach is not applicable to a system in which users have different requested rates. To address this issue, Shen, Andrews, and Evans [3] and Ren *et al.* [4] formulated a nonlinear optimization problem that guarantees proportional rate fairness. However, solving this nonlinear problem can be computationally complex.

To reduce complexity, assigning a group or chunk of adjacent subcarriers in OFDMA was proposed by [5]–[7]. Since a number of channel-filter taps is much lower than that of total subcarriers, adjacent subcarriers are highly correlated. Therefore, if subcarriers are appropriately grouped together and selected for users, the resulting achievable rate can approach that of a single-subcarrier-based allocation at a lower computational cost.

In this paper, we propose a chunk-based subcarrier assignment (SA) algorithm, which is based on [2], [3], but offers a better performance in both single-subcarrier-based and chunk-based resource allocation. We consider a downlink OFDMA in which a base station, which knows all user channels, assigns subcarrier chunks to users to maximize rate requirement, subject to a total power constraint. Rather than assigning subcarrier chunk to the user with the highest channel gain, we propose to assign the chunk to the user with the highest requested rate relative to the average user rate. This idea stems from the well-known proportional fairness scheduling algorithm. Given SA, we propose a simple power allocation (PA) algorithm, which is based on a low signal-to-noise ratio (SNR) approximation. With the approximation, the problem becomes linearized and its solution can be easily obtained. We note that the proposed PA algorithm also works well in a moderate SNR regime.

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II. Channel Model & Problem Formulation

We consider a downlink OFDMA, which consists of a base station and K mobile users. Transmission between the base station and user k for $1 \le k \le K$ is over frequency-selective Rayleigh fading channel with order ℓ_k. All ℓ_k channel-filter taps for user k denoted by $\{h_{k,0}, h_{k,1}, \ldots, h_{k,\ell_k-1}\}$ are independent complex Gaussian random variables with zero mean and variance $1/\ell_k$. Thus, the average gain from channel impulse response of all users is normalized as follows

$$\sum_{i=0}^{\ell_k-1} E|h_{k,i}|^2 = 1. \tag{1}$$

Assuming total N subcarriers used, the frequency response of user k at subcarrier n can be computed from a channel impulse response by discrete Fourier transform (DFT) as follows

$$H_{k,n} = \sum_{i=0}^{\ell_k-1} h_{k,i}\, \mathrm{e}^{\frac{-j2\pi in}{N}}. \tag{2}$$

To transmit data from the base station, each user is assigned a group or multiple groups of contiguous subcarriers. Each group or chunk of subcarriers is exclusively used by a single user, and thus, there is no interference in each subcarrier. Let M be the number of subcarrier chunks, which is greater or equal to the number of users ($M \geq K$). Each chunk contains L subcarriers where $L = N/M$. We assume that N is a multiple of M and thus, L is an integer. Per 3GPP standards for LTE-Advanced, the size of a physical resource block is fixed with 12 subcarriers. Thus, assuming that all chunk sizes are set to L is reasonable.

Signal in each subcarrier is corrupted by additive white Gaussian noise with zero mean and variance σ_w^2. Therefore, an achievable rate for user k is given by

$$R_k = \sum_{m=1}^{M} \omega_{k,m} R_{k,m} \tag{3}$$

where $\omega_{k,m} \in \{0, 1\}$ is an indicator function whose value indicates whether user k is assigned to transmit in chunk m. Since only one user is assigned to transmit in each subcarrier chunk at any moment, for $1 \leq m \leq M$,

$$\sum_{k=1}^{K} \omega_{k,m} = 1. \tag{4}$$

The rate of user k at chunk m per total subcarrier is given by

$$R_{k,m} = \frac{1}{N} \sum_{n=(m-1)L+1}^{mL} \log_2 \left(1 + \frac{p_{k,n}|H_{k,n}|^2}{\sigma_w^2}\right) \tag{5}$$

where $p_{k,n}$ is a transmission power allocated for user k at subcarrier n.

In the end, we would like to maximize the sum achievable rate per subcarrier over chunk assignment and PA with total transmission power for all users P_T. Also, users may have different rate requirements, which affect chunk assignment and PA. Let $\gamma_1 : \gamma_2 : \cdots : \gamma_K$ be a ratio between required rates of all users. Thus, the optimization problem we are interested in solving can be stated as follows:

$$\max_{\{\omega_{k,m}\},\{p_{k,n}\}} \sum_{k=1}^{K} R_k \tag{6}$$

$$\text{subject to} \sum_{k=1}^{K} \sum_{n=1}^{N} p_{k,n} \leq P_T \tag{7}$$

$$p_{k,n} \geq 0, \quad \text{for } 1 \leq k \leq K \text{ and } 1 \leq n \leq N \tag{8}$$

$$\sum_{k=1}^{K} \omega_{k,m} = 1, \quad \text{for } 1 \leq m \leq M \tag{9}$$

$$\omega_{k,m} \in \{0, 1\} \tag{10}$$

$$R_1 : R_2 : \ldots : R_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K. \tag{11}$$

Steps needed to transform this nonconvex problem into a convex optimization problem can be found in [2], [3]. However, finding the solution of a joint SA and PA, which is an integer nonlinear problem, is still very difficult.

III. Proposed Suboptimal Solutions

We propose a suboptimal two-stage solution. First, SA is performed, assuming uniform power allocation for all subcarriers. Then, given SA from the first stage, PA is performed. In general, when the number of chunks is large enough, SA can roughly satisfy users' target rates.

A. Chunk Assignment Algorithm

In [3] and [4], in the first iteration, a subcarrier with the highest channel gain is assigned according to the user's order. This gives favor to the first user, which becomes noticeable with large chunk sizes, i.e. the first user has the highest rate and the last user has the lowest. In addition, assignment based only on the largest channel gain might not result in the largest total throughput. Therefore, our proposed SA removes sequential assignment in the first iteration, and will use both a ratio between the highest rate and the average user rate as an assignment criterion.

In the first iteration, the algorithm assigns each user a chunk of subcarriers by maximizing the ratio between the sum rate in a chunk over the average sum rate in that chunk per user or $\max_{k,m} \frac{R_{k,m}}{\frac{1}{K}\sum_k R_{k,m}}$. Thus, the user with the better channel in a chunk will be assigned that chunk. Since $M \geq K$, each user will have one chunk. Here we apply uniform power across subcarriers, i.e., $p_{k,n} = \frac{P_T}{N}$.

In subsequent iterations, the user with the smallest $\frac{R_k}{\gamma_k}$ will select the chunk that maximizes $\frac{R_{k,m}}{\frac{1}{K}\sum_k R_{k,m}}$ over all the remaining chunks. We iterate until all chunks are assigned. The algorithm is stated in Algorithm 1.

We denote the set of chunks assigned to user k by Θ_k. Thus, after Algorithm 1 is performed. We obtain $\Theta_1, \Theta_2, \cdots, \Theta_K$ for which the union of all sets spans all chunks and no set is overlapping. Let N_k be the number of subcarriers assigned to user k. Therefore, $\sum_{k=1}^{K} N_k = N$.

Algorithm 1 Subcarrier assignment (SA)

Set $S = \{1, 2, \ldots, M\}$.
Set $U, U' = \{1, 2, \ldots, K\}$.
Set $\Theta_k = \emptyset, \forall k \in U$.
Set $R_k = 0, \forall k \in U$.
while $U \neq \emptyset$ **do**
  Find
$$\{k^*, m^*\} = \arg\max_{\substack{k \in U \\ m \in S}} \frac{R_{k,m}}{\frac{1}{K}\sum_{k'' \in U'} R_{k'',m}}.$$
  Update $\Theta_{k^*} \leftarrow \Theta_{k^*} \cup \{m^*\}$ and $S \leftarrow S \setminus \{m^*\}$.
  Update $R_{k^*} \leftarrow R_{k^*} + R_{k^*,m^*}$ and $U \leftarrow U \setminus \{k^*\}$.
end while
while $S \neq \emptyset$ **do**
  Find
$$k' = \arg\min_{k \in U'} \frac{R_k}{\gamma_k}.$$
  Find
$$m' = \arg\max_{m \in S} \frac{R_{k',m}}{\frac{1}{K}\sum_{k \in U'} R_{k,m}}.$$
  Update $\Theta_{k'} \leftarrow \Theta_{k'} \cup \{m'\}$ and $S \leftarrow S \setminus \{m'\}$.
  Update $R_{k'} \leftarrow R_{k'} + R_{k',m'}$.
end while

The number of total iterations in Algorithm 1 equals the number of available chunks M, which is much smaller than the number of subcarriers N. Thus, the proposed algorithm is less complex than non-chunk assignment in [3].

B. Power Allocation Algorithm

We denote the set of subcarriers in all chunks assigned to user k by

$$\Omega_k = \bigcup_{m\in\Theta_k} \bigcup_{n=(m-1)L+1}^{mL} \{n\}. \tag{12}$$

Given SA, the problem in (6) is reduced to

$$\max_{\{p_{k,n}\}} \sum_{k=1}^{K} R_k \tag{13}$$

$$\text{subject to} \sum_{k=1}^{K} \sum_{n\in\Omega_k} p_{k,n} \le P_T \tag{14}$$

$$p_{k,n} \ge 0, \quad \text{for } 1 \le k \le K \text{ and } 1 \le n \le N \tag{15}$$

$$R_1 : R_2 : \ldots : R_K = \gamma_1 : \gamma_2 : \ldots : \gamma_K. \tag{16}$$

A PA problem in (13) was solved by [3]. The solution is as follows. For each subcarrier assignment Ω_k, subcarriers is ordered by a ratio of its squared channel magnitude to the noise power $G_{k,(n)} \triangleq |H_{k,(n)}|^2/\sigma_w^2$ in an increasing order, i.e., $G_{k,(1)} \le G_{k,(2)} \le \cdots \le G_{k,(N_k)}$. The optimal power for user k is then computed by [3]

$$p_{k,(n)} = p_{k,(1)} + \frac{G_{k,(n)} - G_{k,(1)}}{G_{k,(n)}G_{k,(1)}} \tag{17}$$

and a total power allocated to user k is given by

$$P_{T,k} = \sum_{n=1}^{N_k} p_{k,(n)} = N_k p_{k,(1)} + \sum_{n=2}^{N_k} \frac{G_{k,(n)} - G_{k,(1)}}{G_{k,(n)}G_{k,(1)}}. \tag{18}$$

To find the set of optimal total power allocated to all users $\{P_{T,k}\}$, we need to solve the following nonlinear system [3]

$$\frac{N_1}{\gamma_1}\left\{\log_2\left(1 + G_{1,(1)}\frac{P_{T,1} - V_1}{N_1}\right) + \log_2 W_1\right\} = \frac{N_k}{\gamma_k}\left\{\log_2\left(1 + G_{k,(1)}\frac{P_{T,k} - V_k}{N_k}\right) + \log_2 W_k\right\}, \forall k \tag{19}$$

and

$$\sum_{k=1}^{K} P_{T,k} = P_T, \tag{20}$$

where for $2 \le k \le K$,

$$V_k = \sum_{n=2}^{N_k} \frac{G_{k,(n)} - G_{k,(1)}}{G_{k,(n)}G_{k,(1)}} \tag{21}$$

$$W_k = \left(\prod_{n=2}^{N_k} \frac{G_{k,(n)}}{G_{k,(1)}}\right)^{\frac{1}{N_k}}. \tag{22}$$

However, solving (19) and (20) is difficult and requires numerical solutions. To linearize the system in (19) and (20), we apply a low-SNR approximation to obtain a suboptimal PA. As subsequent numerical examples shown, the proposed solution also performs well for a moderate SNR system.

We start with

$$\frac{R_k}{\gamma_k} = \frac{1}{\gamma_k N} \sum_{n_k\in\Omega_k} \log_2(1 + \frac{1}{\sigma_w^2} p_{k,n_k}|H_{k,n_k}|^2) \tag{23}$$

$$\approx \frac{\log_2(\mathrm{e})}{\gamma_k \sigma_w^2 N} \sum_{n_k\in\Omega_k} p_{k,n_k}|H_{k,n_k}|^2 \tag{24}$$

where we assume a low SNR regime, i.e., $P_T/\sigma_w^2 \ll 1$ and apply $\log_2(1+x) \approx x\log_2(\mathrm{e})$ when $x \ll 1$. Using (24) with proportional rate constraint

$$\frac{R_1}{\gamma_1} = \frac{R_2}{\gamma_2} = \cdots = \frac{R_K}{\gamma_K}, \tag{25}$$

and some algebraic manipulation, we obtain a linear system with K equations and K unknowns, which can be written in a matrix equation as follows

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \alpha_2 & 0 & \cdots & 0 \\ 1 & 0 & \alpha_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \cdots & \alpha_K \end{bmatrix} \begin{bmatrix} P_{T,1} \\ P_{T,2} \\ P_{T,3} \\ \vdots \\ P_{T,K} \end{bmatrix} = \begin{bmatrix} P_T \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_K \end{bmatrix} \tag{26}$$

where

$$\alpha_k = -\frac{\gamma_1 E_k N_1 G_{k,(1)}}{\gamma_k E_1 N_k G_{1,(1)}} \tag{27}$$

$$E_k = \sum_{n=1}^{N_k} \frac{G_{k,(n)}}{G_{k,(1)}} \tag{28}$$

and

$$\beta_k = \frac{\gamma_1 E_k N_1}{\gamma_k E_1 G_{1,(1)}} - \frac{\gamma_1 N_1 N_k}{\gamma_k E_1 G_{1,(1)}} - \frac{\gamma_1 E_k N_1 G_{k,(1)}}{\gamma_k E_1 N_k G_{1,(1)}} V_k + \frac{N_1}{G_{1,(1)}}\left(\frac{N_1}{E_1} - 1\right) + V_1. \tag{29}$$

When the solution produced by the linear system (26) is invalid, we revert back to uniform PA. The proposed power allocation algorithm is summarized in Algorithm 2. We note that if $P_{T,k} < V_k$, $p_{k,n} < 0$. To overcome this invalidity, the subcarriers with smallest channel gains will be allocated zero power.

The proposed algorithms greatly reduce computational complexity when compared to solving for the optimal solutions. We note that proposed SA and PA solution obtained from the algorithms do not satisfy the proportional rate constraint. However, as numerical examples will show, the deviation from the rate constraint is relatively small.

IV. Numerical Results

In this section, we show the performance of our proposed SA and PA algorithms and compare them with existing algorithms. Performance index is the minimum user's achievable rate when the rate requirement for all users is the same or a

Algorithm 2 Power allocation (PA)

For each subcarrier assignment Ω_k, obtain $\{G_{k,(n)}\}$.
Determine α_k and β_k for $2 \le k \le K$.
Solve (26) to obtain $\{P_{T,k}\}$.
if $\exists P_{T,k} < 0$ **then**
  $P_{T,k} = \frac{N_k}{N} P_T, \forall k$
end if
while $\exists P_{T,k} < V_k$ **do**
  Update $\Omega_k \leftarrow \Omega_k \setminus \{\arg\min_{n_k \in \Omega_k} |H_{k,n_k}|^2\}$ and $N_k \leftarrow N_k - 1$.
  Update α_k, E_k, V_k, and β_k
  Solve (26) to obtain $\{P_{T,k}\}$.
end while
For each Ω_k, compute $p_{k,(n)}$ from (17) and (18).

sum rate when rate requirements are different. Also how the proposed suboptimal algorithms conform to the proportional rate constraint is measured by average rate-constraint deviation defined by [3]

$$\mathcal{D} = \frac{E\left[\sum_{k=1}^{K}\left|\frac{R_k}{\sum_{k=1}^{K} R_k} - \frac{\gamma_k}{\sum_{k=1}^{K}\gamma_k}\right|\right]}{2 - 2\min_{1\le k\le K}\frac{\gamma_k}{\sum_{k=1}^{K}\gamma_k}} \qquad (30)$$

where expectation is over channel realization. For the optimal solution, $\mathcal{D} = 0$.

A. *Single-Subcarrier-Based Resource Allocation*

Our proposed resource allocation can also be applied with chunk size equal to one and thus, can be directly compared with SA algorithm by [3]. We assume 4 users in a system with the same target rates , i.e. $\gamma_1 : \ldots : \gamma_4 = 1 : \ldots : 1$, and Rayleigh fading channels with 4 taps. To reduce computation time for the optimal solution, which is obtained via a optimization tool, we consider a small system with $N = 16$. (Subsequent examples are shown with a larger N.) In Fig. 1, SA is paired with either our proposed PA stated in Algorithm 2, uniform power in which all subcarriers are allocated equal power, or the optimal PA by [3]. In all combinations, our SA algorithm performs better than Shen *et al.*'s and performs close to the optimal joint SA and PA (up to 98.6%). We remark that the proposed PA, which is based on a low-SNR approximation performs well for all ranges of SNR and that the uniform PA works surprisingly well, but its average rate constraint deviation is worse in a high SNR range shown in Fig. 2.

Fig. 2 shows rate constraint deviation averaged over all users for various algorithms. The optimal PA gives zero deviation and thus, is not shown in the figure. The proposed PA has some rate deviation due to the approximation made to the original problem. We note that the deviation from the proposed PA is in the same order of magnitude as the uniform PA. Generally, for a single-subcarrier-based assignment, uniform PA might suffice since SA are not restricted by subcarrier chunks to satisfy proportional rate requirement.

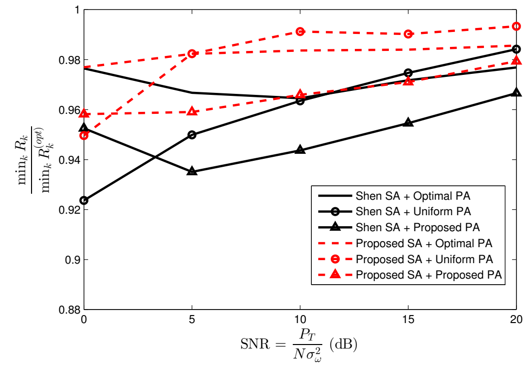

Fig. 1. A fraction of the minimum user's achievable rate ($\min_k R_k$) and the optimal joint SA and PA rate ($\min_k R_k^{(\mathrm{opt})}$) are shown for different algorithms for $K = 4$, $N = 16$, $[\ell_1, \ell_2, \ell_3, \ell_4] = [4, 4, 4, 4]$, and $\gamma_1 : \gamma_2 : \gamma_3 : \gamma_4 = 1 : 1 : 1 : 1$.

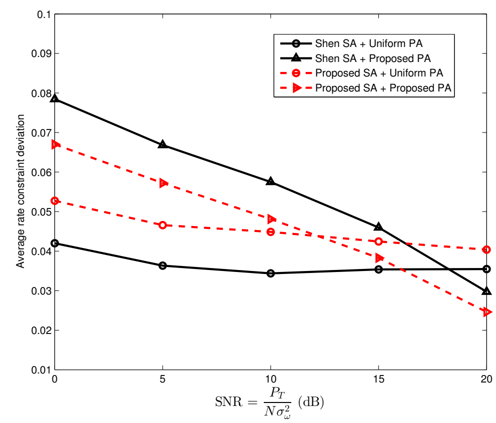

Fig. 2. Average rate-constraint deviation for different algorithms is shown with SNR for $K = 4$, $N = 16$, $[\ell_1, \ell_2, \ell_3, \ell_4] = [4, 4, 4, 4]$ and $\gamma_1 : \gamma_2 : \gamma_3 : \gamma_4 = 1 : 1 : 1 : 1$.

B. *Chunk-Based Resource Allocation*

We now examine performance of the proposed resource allocation with chunk size greater than one. We assume a total of 4 users with equal target rates and the number of available subcarriers is increased to 512. Results are averaged from 10,000 channel realizations. Fig. 3 shows the minimum rate for different chunk sizes for SNR = 20 dB and -5 dB. When chunk size is made larger, the rate is decreased as expected. For 20-dB SNR, it can be seen that the proposed SA outperforms a modified Shen *et al.*'s SA [3] for all chunk sizes. We modify the SA proposed by Shen *et al.* applicable to chunk size greater than one by using sum rate over a chunk in stead of a rate for individual subcarrier.

With uniform PA and the largest chunk size of 128 subcarriers per chunk, the rate obtained from the proposed SA is at 84.4% of the single-subcarrier-based assignment while the rate obtained from [3] is at 79.3% when SNR is 20 dB. We note that the modified SA by [3] is able to achieve a larger rate with the proposed PA than with uniform PA and a lower rate constraint deviation.

For a much lower operating SNR (-5 dB), Fig. 3 show a significant performance gap between uniform PA and the proposed PA. The larger gap is expected as SNR decreases.

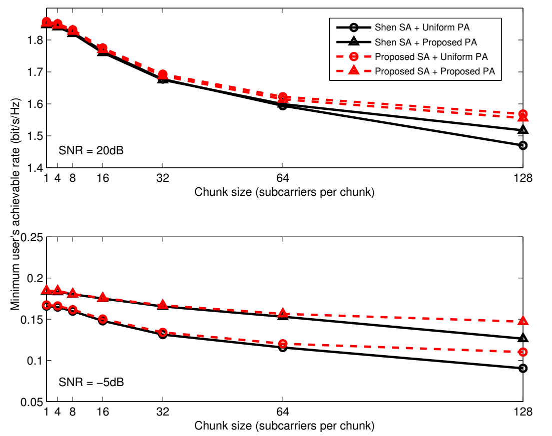

Fig. 3. The minimum user's achievable rate is shown with different chunk size and for both SNR = 20 dB and -5 dB, $K = 4$, $N = 512$, $[\ell_1, \ell_2, \ell_3, \ell_4] = [32, 32, 32, 32]$, and $\gamma_1 : \gamma_2 : \gamma_3 : \gamma_4 = 1 : 1 : 1 : 1$.

Next, users are set to experience different frequency selectivity (different ℓ_k) with two groups of target rates as $2 : 2 : 1 : 1$. A sum rate over all users and the average rate-constraint deviation at SNR 20 dB are shown in Figs. 4 and 5. With a high SNR, sum rate obtained from the proposed SA with uniform PA is the highest for all chunk sizes as seen in Fig. 4. Since the target rates are not the same, proportional rate requirement is not easily achievable for larger chunk size as observed in Fig. 5. To achieve similar rate-constraint deviation as earlier examples with equal rate, selected chunk size has to be less than 32 subcarriers per chunk.

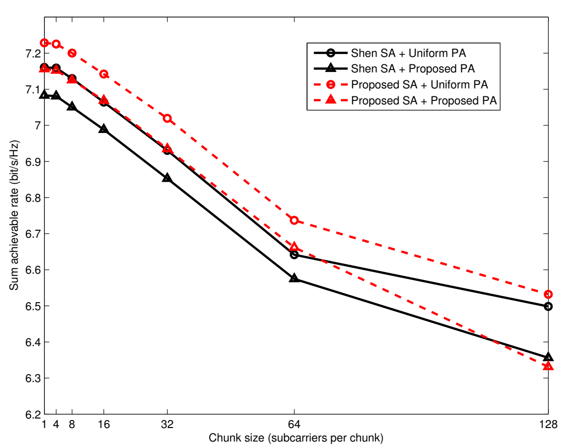

Fig. 4. A sum rate is plotted with chunk size for SNR = 20 dB, $K = 4$, $N = 512$, $[\ell_1, \ell_2, \ell_3, \ell_4] = [4, 8, 16, 32]$, and $\gamma_1 : \gamma_2 : \gamma_3 : \gamma_4 = 2 : 2 : 1 : 1$.

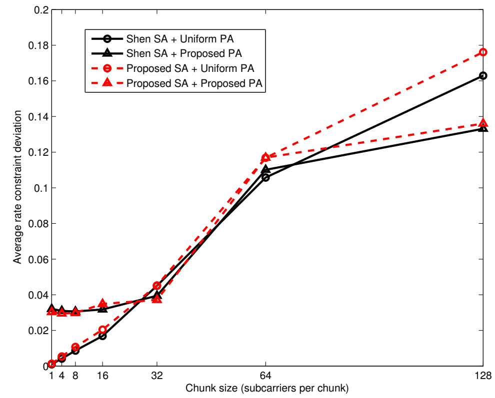

Fig. 5. Average rate-constraint deviation is plotted with chunk size for SNR = 20 dB, $K = 4$, $N = 512$, $[\ell_1, \ell_2, \ell_3, \ell_4] = [4, 8, 16, 32]$, and $\gamma_1 : \gamma_2 : \gamma_3 : \gamma_4 = 2 : 2 : 1 : 1$.

V. Conclusions

In this paper, we propose SA and PA algorithms for a downlink OFDMA with proportional rate constraints. Numerical results show that our proposed SA outperforms existing work by [3] and its modified chunk-based version. In a low SNR regime, user rate is more sensitive to PA and is at its highest with our PA. However, in a high SNR regime, user rate is more sensitive to SA while different PA's do not differ much as expected.

With single-subcarrier assignment, uniform PA is sufficient since the proposed SA mostly satisfies rate requirements. However, with larger subcarrier chunks, proportional rates are more difficult to maintain with only SA. Thus, the proposed PA is needed.

References

[1] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 2, pp. 171–178, Feb 2003.

[2] W. Rhee and J. M. Cioffi, "Increasing in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE Vehicular Technol. Conf. (VTC2000-Spring)*, vol. 2, Tokyo, Japan, 2000, pp. 1085–1089.

[3] Z. Shen, J. G. Andrews, and B. L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2726–2737, Nov 2005.

[4] Z. Ren, S. Chen, B. Hu, and W. Ma, "Proportional resource allocation with subcarrier grouping in ofdm wireless systems," *IEEE Commun. Lett.*, vol. 17, no. 5, pp. 868–871, May 2013.

[5] P. Svedman, S. K. Wilson, L. J. Cimini, and B. Ottersten, "A simplified opportunistic feedback and scheduling scheme for OFDM," in *Proc. IEEE Vehicular Technol. Conf. (VTC2004-Spring)*, vol. 4, Milan, Italy, May 2004, pp. 1878–1882.

[6] H. Zhu and J. Wang, "Chunk-based resource allocation in OFDMA systems–Part I: Chunk allocation," *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2734–2744, Sep 2009.

[7] ——, "Chunk-based resource allocation in OFDMA systems–Part II: Joint chunk, power and bit allocation," *IEEE Trans. Commun.*, vol. 60, no. 2, pp. 499–509, Feb 2012.