Rate-Maximizing User Selection for Downlink NOMA With Effect of Log-Normal Shadowing

Jeong woo Kwon, Wiroonsak Santipach, and Srijidtra Mahapakulchai
Department of Electrical Engineering
Faculty of Engineering, Kasetsart University
Bangkok 10900, Thailand
Email: chonghoo@msn.com, {wiroonsak.s, fengsjm}@ku.ac.th

***Abstract*—We propose binary-search user selection that maximizes sum transmission rate in downlink non-orthogonal multiple access (NOMA). The selected existing user shares a downlink channel with a new cell-edge user and applies successive interference cancellation (SIC) to detect the transmitted symbols. The proposed binary search is shown to be significantly less complex than exhaustive search, especially in a large system. If random shadowing exists in the cell, the base station may not select the optimal user and the rate performance can suffer greatly. Simulation and analytical results for log-normal shadowing are also shown.**

I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has been incorporated in the fifth generation (5G) wireless standards, which are expected to provide very high data rate with low latency as well as achieve high spectral and power efficiency [1], [2]. Unlike orthogonal multiple access (OMA), NOMA allows some interference among users in frequency or time domains, but distinguishes users either in power or code domains. One of the problems for a base station in NOMA is how to determine a set of users to share a downlink channel. In [3], power allocation for random user pairing is analyzed with the condition that the resulting capacity will be at least equal to that in OMA. In [4], user selection and power allocation for orthogonal frequency-division multiplexing (OFDM) NOMA are considered. To lessen search complexity, users who do not satisfy the derived condition, are removed from selection pool. Reference [5] also considers power allocation scheme with user selection for downlink NOMA, but under imperfect channel state information (CSI). Upper bounds for outage probability are derived in closed-form expressions.

In this work, we consider user selection problem in downlink NOMA with a constraint on rate reduction of the existing user, which was not considered in the previous work mentioned. In [6][1], we have proposed binary-search user selection that maximizes sum rate between the existing and the new users who share the same orthogonal channel. If the base station has perfect channel information, the proposed algorithm selects the optimal existing user with much less complexity than exhaustive search. We also extend the work in [6] by examining the effect of log-normal shadowing on the rate performance. If the base station only have the average and not instantaneous channel gains from all users, the optimal user may not be selected and the sum rate can suffer heavily. With a constraint on outage probability, sum rate is shown to decrease linearly with small increase in standard deviation of log-normal shadowing.

II. SYSTEM MODEL

We consider a single-cell downlink in which user signals are transmitted from a base station to maximum M orthogonal mobile users. Assuming large frequency-reuse factor, intercell interference is negligible. The base station is assumed to be at the center of a circular cell with radius of R. For a discrete-time channel model, the received signal for the mth mobile user is given by

$$r_m = \sqrt{g_{E,m} P_T} \cdot s + n_m \tag{1}$$

where s is the transmitted symbol from the base station with zero mean and unit variance, P_T is the transmitted power per user, $g_{E,m}$ is a random channel gain from the base station to user m with mean $\bar{g}_{E,m}$, and n_m is additive white Gaussian noise with zero mean and variance σ_n^2. We assume that the channel fades slowly and degrades largely due to the distance away from the transmitter. Thus, the mean channel gain $\bar{g}_{E,m}$ decreases with user m's distance from the base station. The achievable transmission rate for user m is given by

$$R_{E,m} = W \log_2 \left(1 + \frac{g_{E,m} P_T}{\sigma_n^2} \right) \tag{2}$$

where W is channel bandwidth for each user.

When the number of existing users is less than or equal to M, both the base station and users operate in OMA mode. However, if the number of users is greater than M, we assume that all nodes are capable of operating in NOMA mode as well. If there are M existing orthogonal users in the cell and there is a new user, the base station must assign the new user one of the M channels to be shared with the existing user. In [6], we propose user selection algorithm, which will be described in Section III.

In NOMA mode, the base station applies superposition coding along with channel-inversion power allocation to transmit

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[1]Since the article is in Thai, contributions from [6] are also published here for wider dissemination.

symbols for both existing user m and the new user, which share the same channel. Let g_N denote channel gain for the new user with mean $\bar{g}_N$. In this study, we assume that the new user is on the cell edge and hence, $\bar{g}_N \le \bar{g}_{E,m}, \forall m$. Also the base station only knows the mean channel gains of the users, but not the actual instantaneous values. It can be straightforwardly shown that transmit power for user m and the new user are given by

$$P_{E,m} = \left(\frac{\bar{g}_N}{\bar{g}_N + \bar{g}_{E,m}}\right) 2P_T, \tag{3}$$

$$P_N = \left(\frac{\bar{g}_{E,m}}{\bar{g}_N + \bar{g}_{E,m}}\right) 2P_T. \tag{4}$$

Since $\bar{g}_N \le \bar{g}_{E,m}$, $P_N \ge P_{E,m}$ and $P_{E,m} \le P_T$.

At the receiver of user m, successive interference cancellation (SIC) is applied. The stronger symbol for the new user is decoded first and is subtracted from the received signal. Then, the symbol for user m is decoded. The achievable rate for user m with SIC is given by

$$R_{E,m}^{\text{SIC}} = W \log_2 \left(1 + \frac{g_{E,m} P_{E,m}}{\sigma_n^2}\right). \tag{5}$$

Since $P_{E,m} \le P_T$, the rate of user m when sharing the channel with the new user ($R_{E,m}^{\text{SIC}}$) is less than the rate without any sharing ($R_{E,m}$). The rate reduction for the existing user m must not exceed certain threshold ϵ as follows

$$\frac{R_{E,m} - R_{E,m}^{\text{SIC}}}{R_{E,m}} \le \epsilon \tag{6}$$

where $0 < \epsilon < 1$.

For the new user, the received symbol for user m is much weaker and only marginally interferes with the symbol for the new user. Thus, there is no need to apply SIC. Hence, the achievable rate for the new user is given by

$$R_N = W \log_2 \left(1 + \frac{g_N P_N}{g_N P_{E,m} + \sigma_n^2}\right) \tag{7}$$

where there is some interference from the symbol for user m.

III. Proposed User Selection

We propose to select the existing user m that gives the maximum sum rate with the new user, subject to a constraint on the rate reduction as follows

$$\begin{aligned} m^* &= \arg\max_m R_{E,m}^{\text{SIC}} + R_N \\ \text{subject to} \quad & R_{E,m^*}^{\text{SIC}} \ge (1-\epsilon) R_{E,m^*}. \end{aligned} \tag{8}$$

If the base station knows the actual channel gains of all users instead of the means, the solution to the problem (8) can be derived as follows. First, transmit power for the existing user m and the new user in (3) and (4) is computed with the actual gains $g_{E,m}$ and g_N instead of the mean values $\bar{g}_{E,m}$ and $\bar{g}_N$. It is straightforward to show that the sum rate $R_{E,m}^{\text{SIC}} + R_N$ is monotonically increasing with $g_{E,m}$. We note that if there was no constraint on the rate reduction (8), the base station would pick the user with the largest channel gain, which is the closest to the base station.

Substituting (2) – (5) into the rate constraint in (8) with the assumption that the base station knows the actual channel gains, the constraint can be expressed as follows

$$g_N \ge g_{E,m} \left(\frac{1}{1 - \frac{\sigma_n^2}{2P_T}\left[\left(1 + \frac{g_{E,m} P_T}{\sigma_n^2}\right)^{1-\epsilon} - 1\right]} - 1 \right). \tag{9}$$

Thus, the solution to the problem (8) is to select user m with the largest channel gain $g_{E,m}$ that also satisfies (9). To search for that user, the base station can order all channel gains $g_{E,m}, \forall m$ from the largest to the smallest and check each gain with (9). The ordered gains are denoted by $g_{E,(1)} \ge g_{E,(2)} \ge \cdots \ge g_{E,(M)}$. Thus, search complexity linearly increases with M. To reduce the complexity of user selection, we propose a binary search, which is detailed in Algorithm 1.

Algorithm 1 The proposed binary search for user selection.

Require: $g_{E,1}, g_{E,2}, \cdots, g_{E,M}$, g_N, P_T/σ_n^2, and ϵ

Sort $\{g_{E,m}\}$ from the largest to the smallest as follows $g_{E,(1)} \ge g_{E,(2)} \ge \cdots \ge g_{E,(M)}$.
Set `start` $\leftarrow 1$ and `stop` $\leftarrow M$
if $g_{E,(\texttt{start})}$ satisfies (9) **then**
  `selected` $\leftarrow$ `start`
else if $g_{E,(\texttt{start})}$ and $g_{E,(\texttt{end})}$ do not satisfy (9) **then**
  `selected` $\leftarrow$ NULL
else
  `next` $\leftarrow$ floor $\left(\frac{1}{2}(\texttt{start} + \texttt{end})\right)$
  while `end` $-$ `start` ≥ 1 **do**
    if `end` $-$ `start` $= 1$ **then**
      `selected` $\leftarrow$ `next`
      `start` $\leftarrow$ `next`
    end if
    if $g_{E,(\texttt{next})}$ satisfies (9) **then**
      `end` $\leftarrow$ `next`
    else
      `start` $\leftarrow$ `next`
    end if
    `next` $\leftarrow$ floor $\left(\frac{1}{2}(\texttt{start} + \texttt{end})\right)$
  end while
end if
return `selected`

If $g_{E,(1)}$ satisfies (9), but $g_{E,(M)}$ does not, the algorithm determines the gain in the middle (or next to the middle) whether it satisfies (9). If it does not, we keep only the first half of the set of ordered gains and discard the second half. Otherwise, we keep the second half and discard the first half. Then, we determine (9) for the gain in the middle (or next to the middle) of the chosen set. At each iteration, half of the gains will be dropped from the selection pool. We iterate until the solution is found. This binary search can be significantly less complex than exhaustive search when M is very large. This advantage is shown by simulation results in Table I in Section V. We remark that the proposed algorithm does obtain the optimal user selection, which is the solution of (8).

IV. Effect of Log-Normal Shadowing

If the base station does not know the instantaneous channel gains $g_{E,m}$, then the user selection in Algorithm 1 will instead be based on the mean channel gains $\bar{g}_{E,m}$. The selected user m^* may neither gives the maximum sum rate nor satisfies the rate constraint in (8). Suppose the base station transmits symbols for user m^* at the rate of R_{E,m^*}, which is based on the mean channel gains. Outage occurs when channel capacity is less than R_{E,m^*}. The associated outage probability is given by

$$P_{E,m^*}^{\text{out}} = \Pr\left\{ W\log_2\left(1+\frac{1}{\sigma_n^2} g_{E,m^*}P_{E,m^*}\right) < R_{E,m^*}\right\} \tag{10}$$

$$= F_{g_{E,m^*}}\left((2^{R_{E,m^*}/W}-1)\frac{\sigma_n^2}{P_{E,m^*}}\right) \tag{11}$$

where $F_{g_{E,m^*}}(\cdot)$ denotes cumulative distribution function (cdf) for g_{E,m^*}.

We assume that random variation of channel gain is due to blockage from objects in the signal path, which is called shadowing, and follows log-normal distribution. Consequently, channel gain g_{E,m^*} in decibel (dB) is modeled as Gaussian random variable with mean $\bar{g}_{E,m^*}$ in dB and variance σ_s^2 in dB squared. Outage probability with log-normal shadowing for selected user m^* is given by

$$P_{E,m^*}^{\text{out}} = \frac{1}{2}+\frac{1}{2}\operatorname{erf}\Big(\big(\ln(2^{R_{E,m^*}/W}-1) - \ln(P_{E,m^*}/\sigma_n^2) - \mu_{E,m^*}\big)/(0.1\sqrt{2}\ln(10)\sigma_s)\Big) \tag{12}$$

where $\operatorname{erf}(\cdot)$ denotes the error function and $\mu_{E,m^*} = 0.1\ln(10)\bar{g}_{E,m^*}(\text{dB})$. As shadowing intensifies (σ_s increases), outage probability as shown by (12) also increases since the error function is an increasing function. If we wish to constrain outage probability not to exceed certain threshold δ ($P_{E,m^*}^{\text{out}} \le \delta$), it can be shown from (12) that the transmission rate must not exceed the rate bound given by

$$R_{E,m^*} \le W\log_2\left(1+\frac{\bar{g}_{E,m^*}P_{E,m^*}}{\sigma_n^2}10^{0.1\sqrt{2}\sigma_s \operatorname{erf}^{-1}(2\delta-1)}\right) \tag{13}$$

where $\operatorname{erf}^{-1}(\cdot)$ is the inverse error function.

We can also derive outage probability for the new user denoted by P_N^{out}. If the same constraint on outage probability is enforced, the transmission rate for the new user must not exceed the following rate bound

$$R_N \le W\log_2\left(1+\frac{\bar{g}_N P_N}{\bar{g}_N P_{E,m^*}+\sigma_n^2 10^{0.1\sqrt{2}\sigma_s \operatorname{erf}^{-1}(2\delta-1)}}\right). \tag{14}$$

From rate constraints (13) and (14), both transmission rates must be reduced when outage threshold δ decreases. If $\delta \to 0$, both rates must converge to zero as well. Also note that when shadowing diminishes ($\sigma_s \to 0$), the rate bounds in (13) and (14) are converging to the transmission rates without shadowing in (2) and (7), respectively.

When signal-to-noise ratio (SNR) is large ($P_T/\sigma_n^2 \gg 1$) and variance of log-normal shadowing is small, we can approximate the rate bound for selected user m^* in (13) as a linear function given by

$$R_{E,m^*} \lesssim W\log_2\left(\frac{\bar{g}_{E,m^*}P_{E,m^*}}{\sigma_n^2}\right) + 0.1\log_2(10)\sqrt{2}W\sigma_s \operatorname{erf}^{-1}(2\delta-1). \tag{15}$$

The first term of the approximate rate bound (15) is the approximate rate without shadowing. As shadowing becomes more severe (larger σ_s but still close to zero), the rate bound linearly decreases. If the cell is also large, the average interference power to the new user $\bar{g}_N P_{E,m^*}$ is small. With large-cell assumption, we can derive the approximate rate bound for the new user, which is the same as (15) except that $\bar{g}_{E,m^*}P_{E,m^*}$ is replaced by $\bar{g}_N P_N$. Thus, with those assumptions, the rate for the new user has to decrease linearly with σ_s as well.

V. Numerical Results

For the first simulation result, we assume that there are $M = 1000$ existing users in the cell with 100-meter radius. Each existing user shown by a dot in Fig. 1, is assumed to be uniformly placed in a circular cell with the base station at its center. We assume that there is no shadowing ($\sigma_s = 0$) and the channel gain from the base station to each user is inversely proportional to the squared distance. With placement of the new user shown in the figure, we apply the proposed binary search in Algorithm 1 to locate the optimal existing user. With $\epsilon = 0.05$, 0.1, and 0.2, the selected users are shown in the figure in red circles. We see that the larger the rate reduction the existing user can endure, the closer the selected user is to the base station. If the rate constraint did not exist, the selected user that maximizes sum rate would be the user closest to the base station.

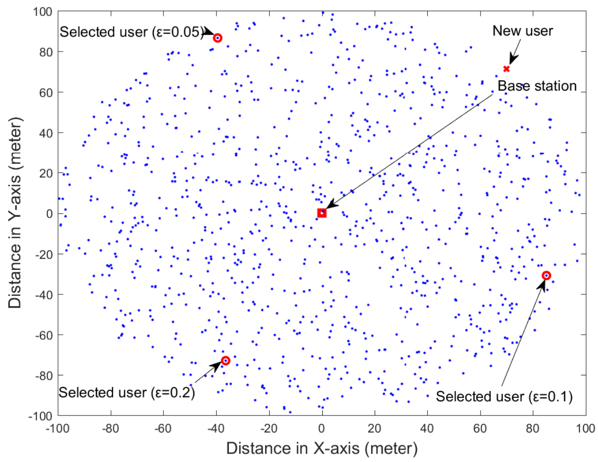

Fig. 1. A circular cell is shown with 1000 uniformly placed users. and the new user on the cell edge. The selected user to share the channel with the new user is shown for various values of ϵ and 60-dB SNR (P_T/σ_n^2).

Next we examine the search complexity by counting the number of comparisons (9) the algorithm needs to execute, and show the result in Table I. As order of magnitude of the number of existing users (M) increases, the average number of comparisons (9) performed only increases linearly. This is in sharp contrast to exhaustive search for which complexity is linearly proportional to M.

TABLE I
COMPLEXITY OF THE PROPOSED USER SELECTION

Number of existing users (M)	**Average number of comparisons (9) performed**
100	6.66
1,000	9.97
10,000	13.36

In Fig. 2, we compare sum rate of the new user and the selected existing user in NOMA mode with that in OMA mode. For NOMA, the existing user requires SIC and the base station must apply superposition coding and channel-inversion power allocation as described in Section II. For OMA, the selected user must give ϵ fraction of its bandwidth to the new user instead of sharing the entire bandwidth. From Fig. 2, we see that OMA is inferior to NOMA for both SNR at 80 and 90 dB. When $\epsilon = 0.1$ and SNR is 80 dB, sum rate of OMA is less than half of NOMA. However, for 90-dB SNR, the rate difference is not as pronounced. We note that as ϵ increases or the rate constraint becomes more relaxed, the sum rate generally increases.

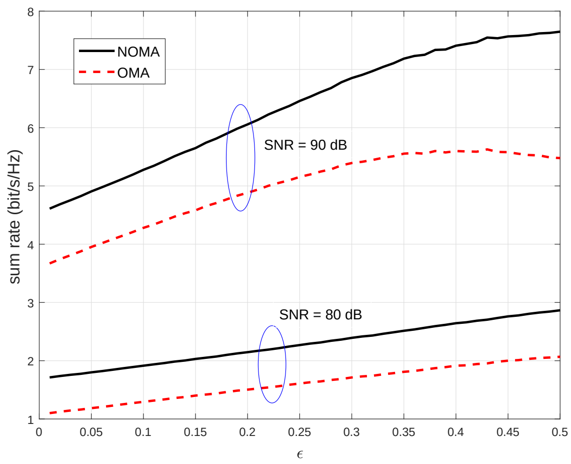

Fig. 2. Sum rates of the selected existing user and the new user in NOMA and OMA modes are compared for various ϵ.

The effect of log-normal shadowing is illustrated in Fig. 3 in which the transmission rate is shown with σ_s in dB for different outage requirement. When $\sigma_s = 0$ (no shadowing), the algorithm 1 performed by the base station will select the optimal existing user. As shadowing becomes worsen, the selected user may not be close to the optimal user that maximizes sum rate. Also, as required outage probability δ decreases from 20% to 10% and 5%, transmission rate is reduced accordingly. We observe linear rate reduction when σ_s is small as predicted by analysis in (15). For 5%-outage, we also show the transmission rate for the new user marked by squares in the figure and that for the existing user marked by solid line. We see that both decrease at similar rate with weak shadowing (small σ_s). From the figure, we observe that even weak shadowing can have adverse effect on the rate performance.

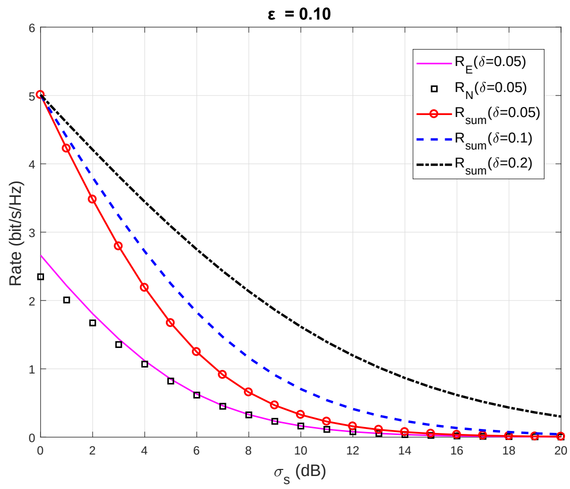

Fig. 3. Transmission rates for existing user and new user, and sum rate are shown with σ_s, outage probability of 0.05, 0.1, and 0.2, and ϵ set to 0.1.

VI. CONCLUSIONS

The proposed binary-search user selection can find the optimal existing user to be paired with the new user in downlink NOMA. The complexity of the proposed scheme only increases linearly when the number of users in the cell to be searched increases exponentially. However, if the base station only have the average channel gains instead of the actual values, the sum rate can suffer significantly. With log-normal shadowing, sum rate decreases linearly with small increase in standard deviation of the shadowing. For future work, multicell model with fractional frequency reuse or small-scale fading can be considered.

REFERENCES

[1] F. Luo and C. Zhang, *Signal Processing for 5G: Algorithms and Implementations*. John Wiley & Sons, 2016, pp. 143–168.

[2] M. Agiwal, A. Roy, and N. Saxena, "Next generation 5G wireless networks: A comprehensive survey," *IEEE Commun. Surveys Tuts.*, vol. 18, no. 3, pp. 1617–1655, third quarter 2016.

[3] J. A. Oviedo and H. R. Sadjadpour, "A fair power allocation approach to NOMA in multiuser SISO systems," *IEEE Trans. Veh. Technol.*, vol. 66, no. 9, pp. 7974–7985, Sep. 2017.

[4] G. Nain, S. S. Das, and A. Chatterjee, "Low complexity user selection with optimal power allocation in downlink NOMA," *IEEE Wireless Commun. Lett.*, vol. 7, no. 2, pp. 158–161, Apr. 2018.

[5] W. Cai, C. Chen, L. Bai, Y. Jin, and J. Choi, "User selection and power allocation schemes for downlink NOMA systems with imperfect CSI," in *Proc. IEEE Veh. Technol. Conf. (VTC-Fall)*, Montreal, QC, Canada, Sep. 2016, pp. 1–5.

[6] J. W. Kwon, C. Sriwichien, and W. Santipach, "Optimal user selection with binary search for downlink NOMA (in Thai)," in *Proc. 41st Elect. Eng. Conf. (EECON)*, Ubonratchathani, Thailand, Nov. 2018, pp. 1–4.