Chapter 4

Math Model

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Chapter 4

Mathematic Model

4.0 Type of Data

4.1 Linear Equation

4.2 Linear Inequalities

4.3 Solution of two linear inequality

4.4 Linear programming

4.5 Non-linear Model

4.0 Type of Data

⇒1. Qualitative data

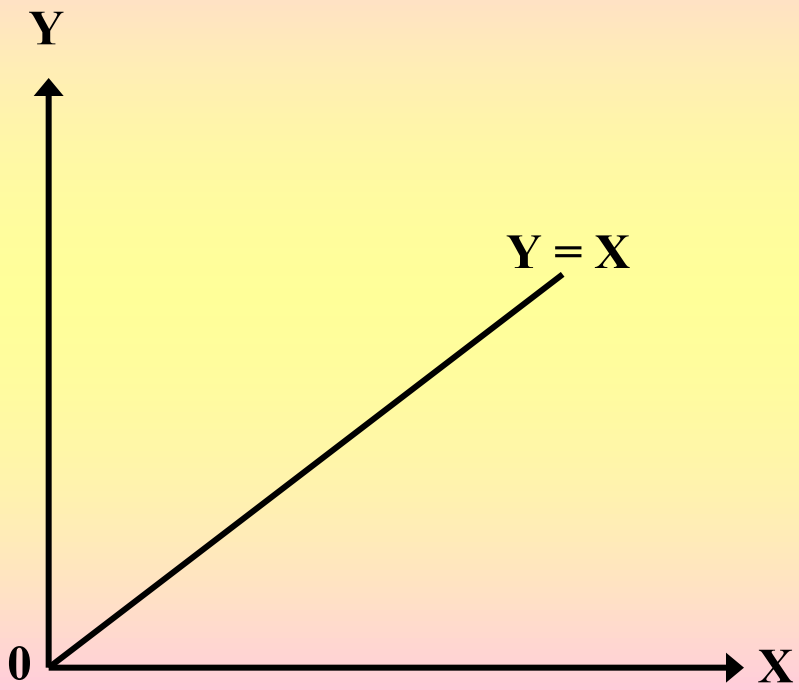
- Deals with descriptions.
- Data can be observed but not measured.
- Colors, textures, smells, tastes, appearance, beauty, etc.

⇒2. Quantitative data

- Deals with numbers.
- Data which can be measured.
- Length, height, area, volume, weight,

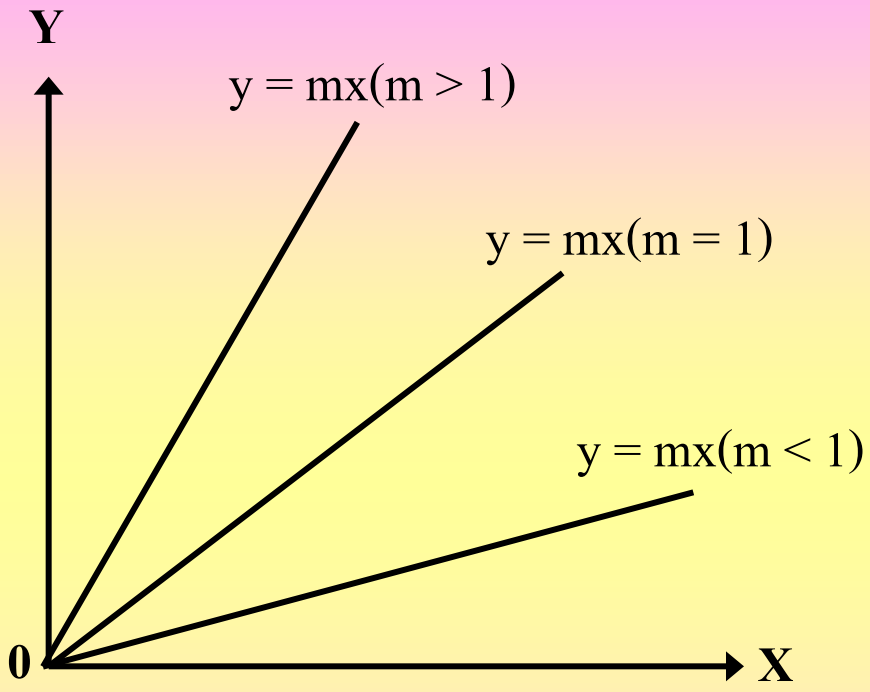
4.1 Linear Equation

1. Y = X



Y = X is the line that starts at the origin (0, 0) and goes up at a 45 degree angle

2. Y = mX



Y = mx,

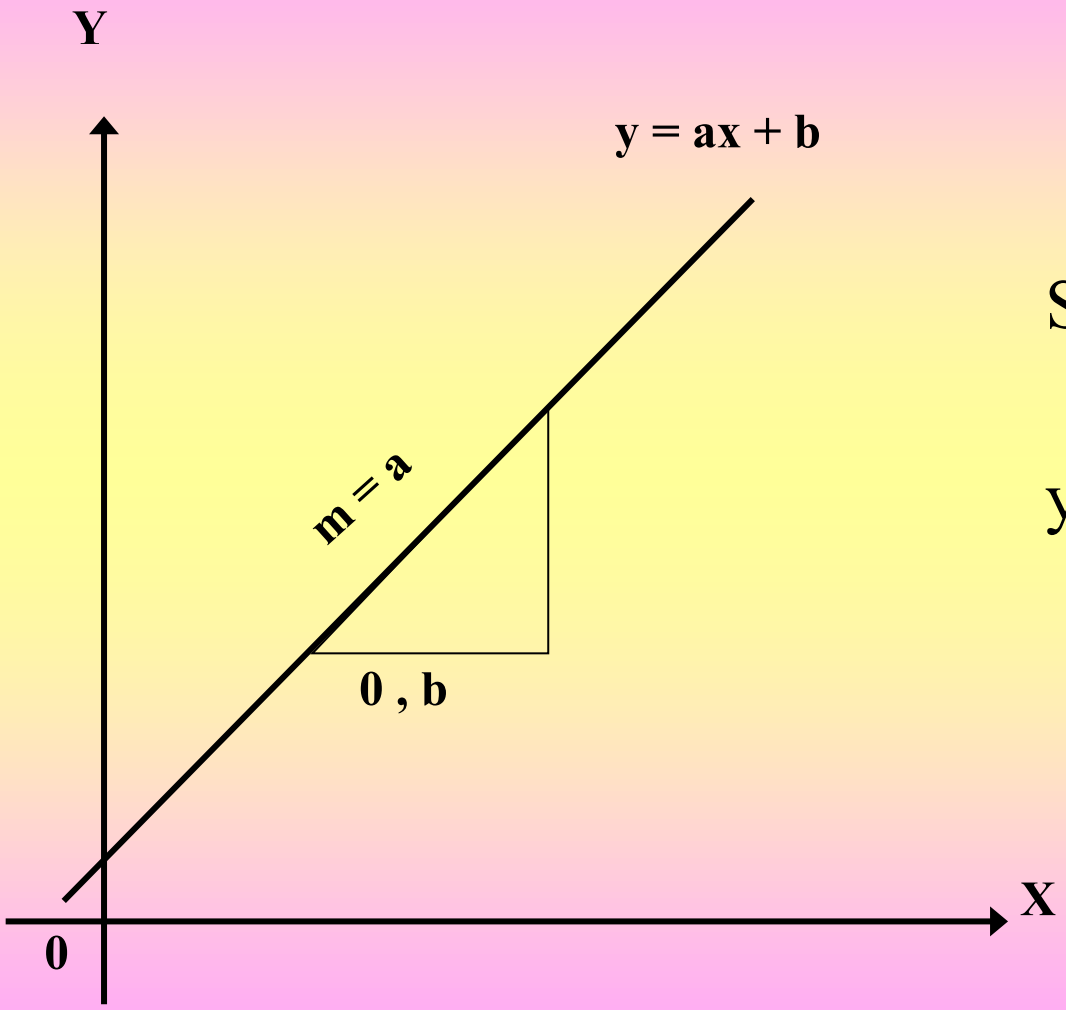
where, m = 1 = slope

Line goes up at a 45 degree angle

m > 1, line is steeper than y = x

m < 1, line is flatter than y = x

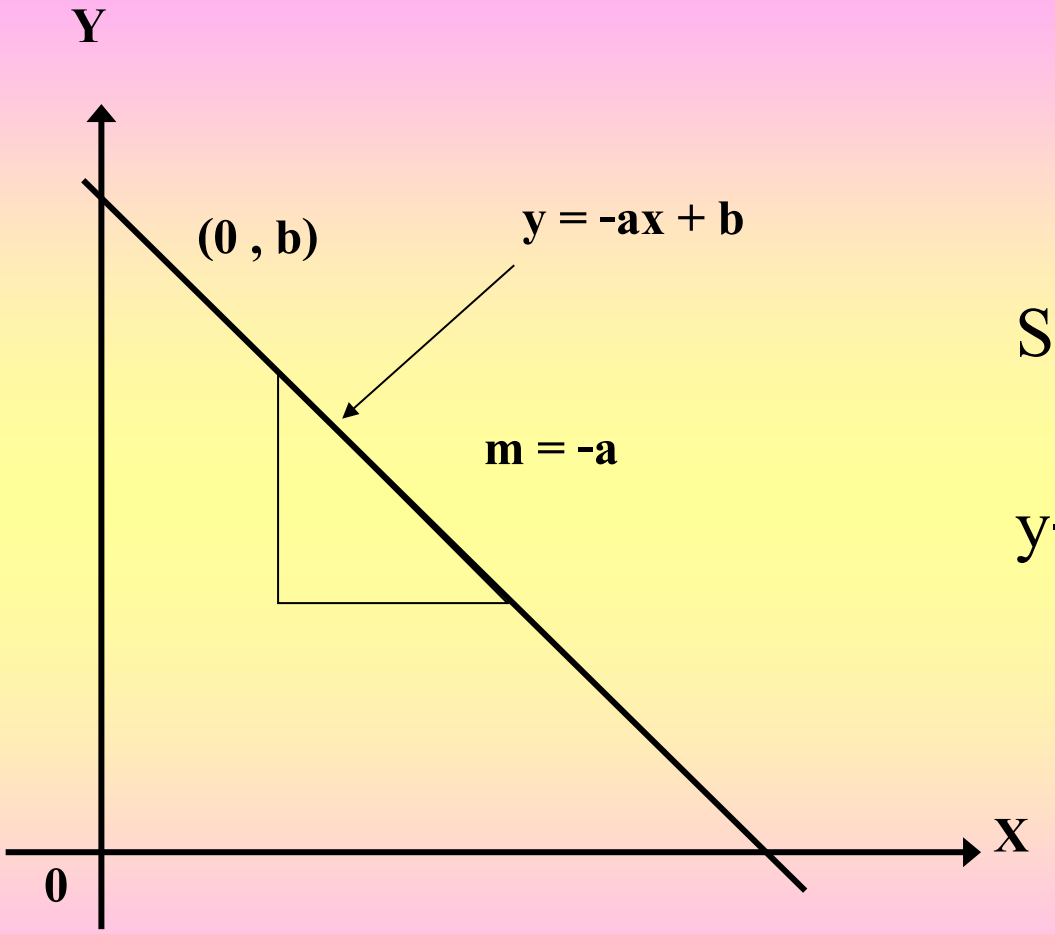
3. Y = aX+b



Slope = a,

y-intercept = b

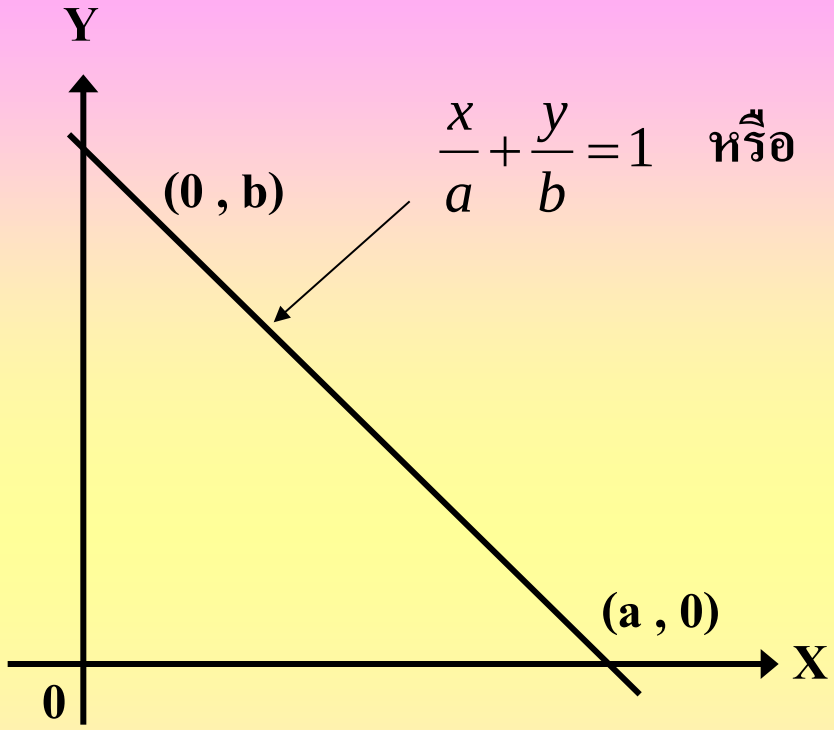
4. Y = -aX+b



Slope = -a,

y-intercept = b

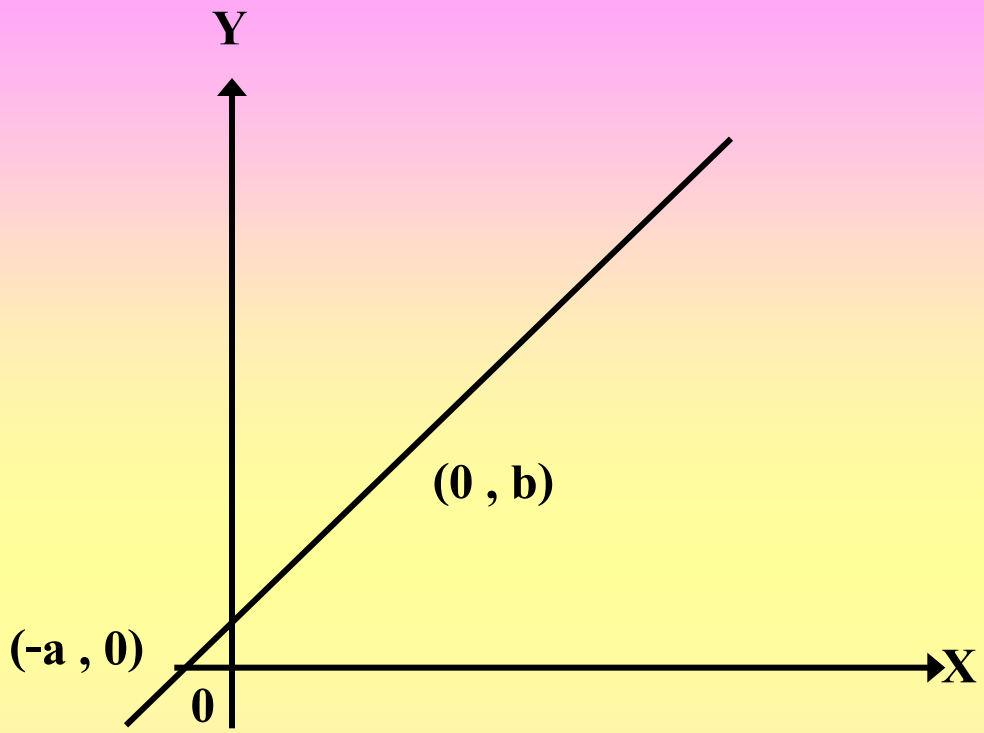
4. Y = -aX+b



$$\frac{x}{a}+\frac{y}{b}=1 \quad \text{หรือ} \quad \frac{bx}{a}+y=b \quad y=\frac{-bx}{a}+b$$

X-intercept at (a , 0) Y-intercept at (0 , b)

slope = –b/a



$$\frac{x}{-a}+\frac{y}{b}=1$$

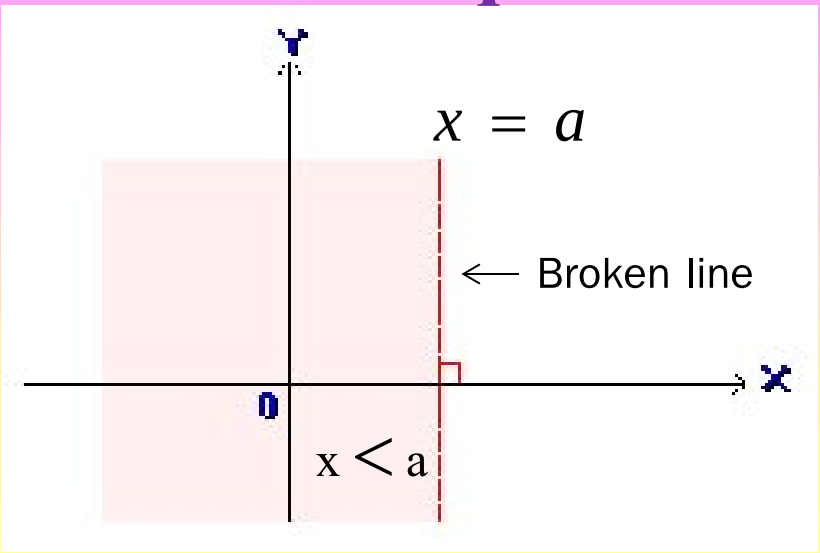
$$\frac{y}{b}=\frac{x}{a}+1$$

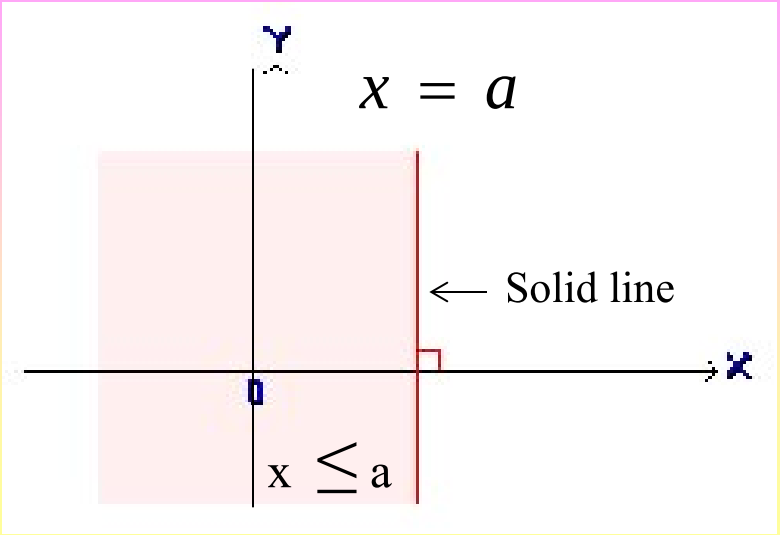
$$y=\frac{bx}{a}+b$$

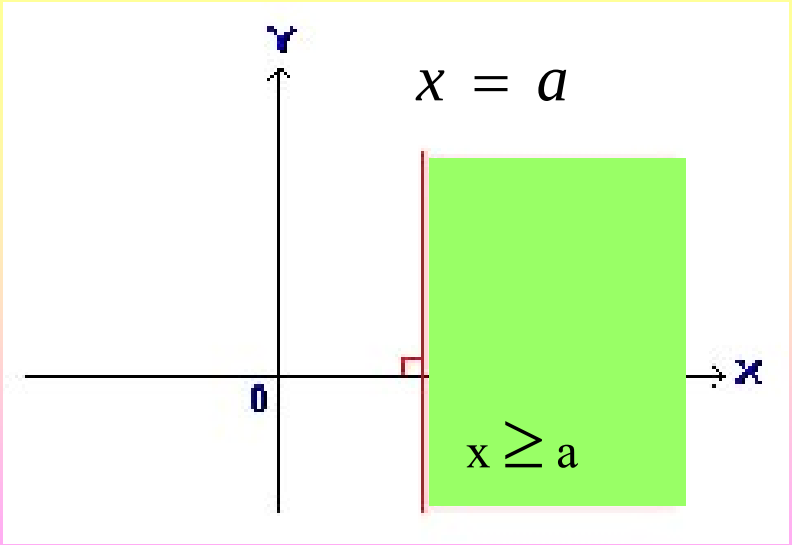
X-intercept at (-a , 0) Y-intercept at (0 , b)

slope = b/a

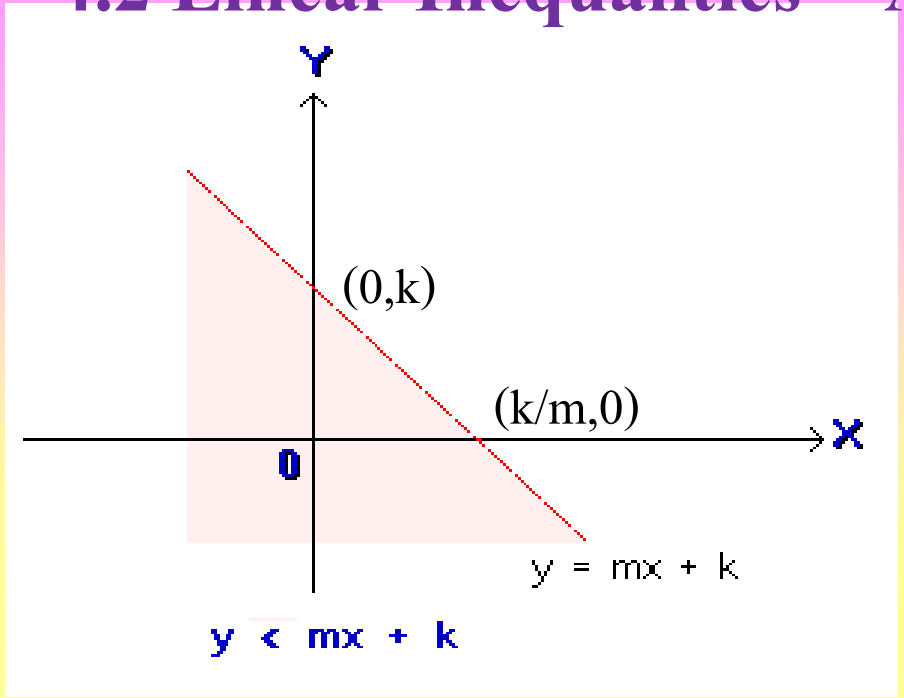
4.2 Linear Inequalities

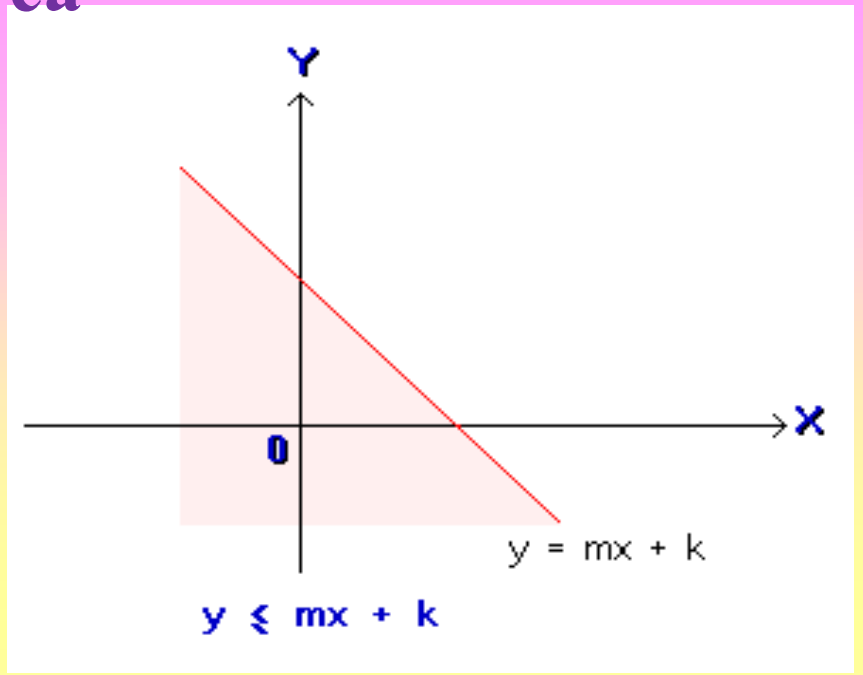






4.2 Linear Inequalities = Area





4.3 Solution of two linear inequality

4.4 Linear programming

Mathematical programming is used to find the best or optimal solution to a problem that requires a decision or set of decisions about how best to use a set of limited resources to achieve a state goal of objectives.

Linear programming requires that all the mathematical functions in the model be linear functions.

4.4 Linear programming

Steps involved in mathematical programming

- Conversion of stated problem into a mathematical model that abstracts all the essential elements of the problem.
- Exploration of different solutions of the problem.
- Finding out the most suitable or optimum solution.

Ex.1 Minimum Cost Problem

		Productivity (tons/day)		
	Mine	Tin	Copper	Zinc
x^1	A	6	2	4
x^2	B	2	2	12

$\frac{days}{week}$

Condition of Production (order)

Tin	12	tons/week
Copper	8	tons/week
Zinc	24	tons/week

$$6x_1 + 2x_2 \geq 12$$

$$2x_1 + 2x_2 \geq 8$$

$$4x_1 + 12x_2 \geq 24$$

$$\frac{x_1}{2} + \frac{x_2}{6} \geq 1$$

$$\frac{x_1}{4} + \frac{x_2}{4} \geq 1$$

$$\frac{x_1}{6} + \frac{x_2}{2} \geq 1$$

Target

Cost of mine A 40,000 baht/day

Cost of mine B 32,000 baht/day

Minimum Cost Equation

Min Cost = $40{,}000\ x_1 + 32{,}000\ x_2$

= min

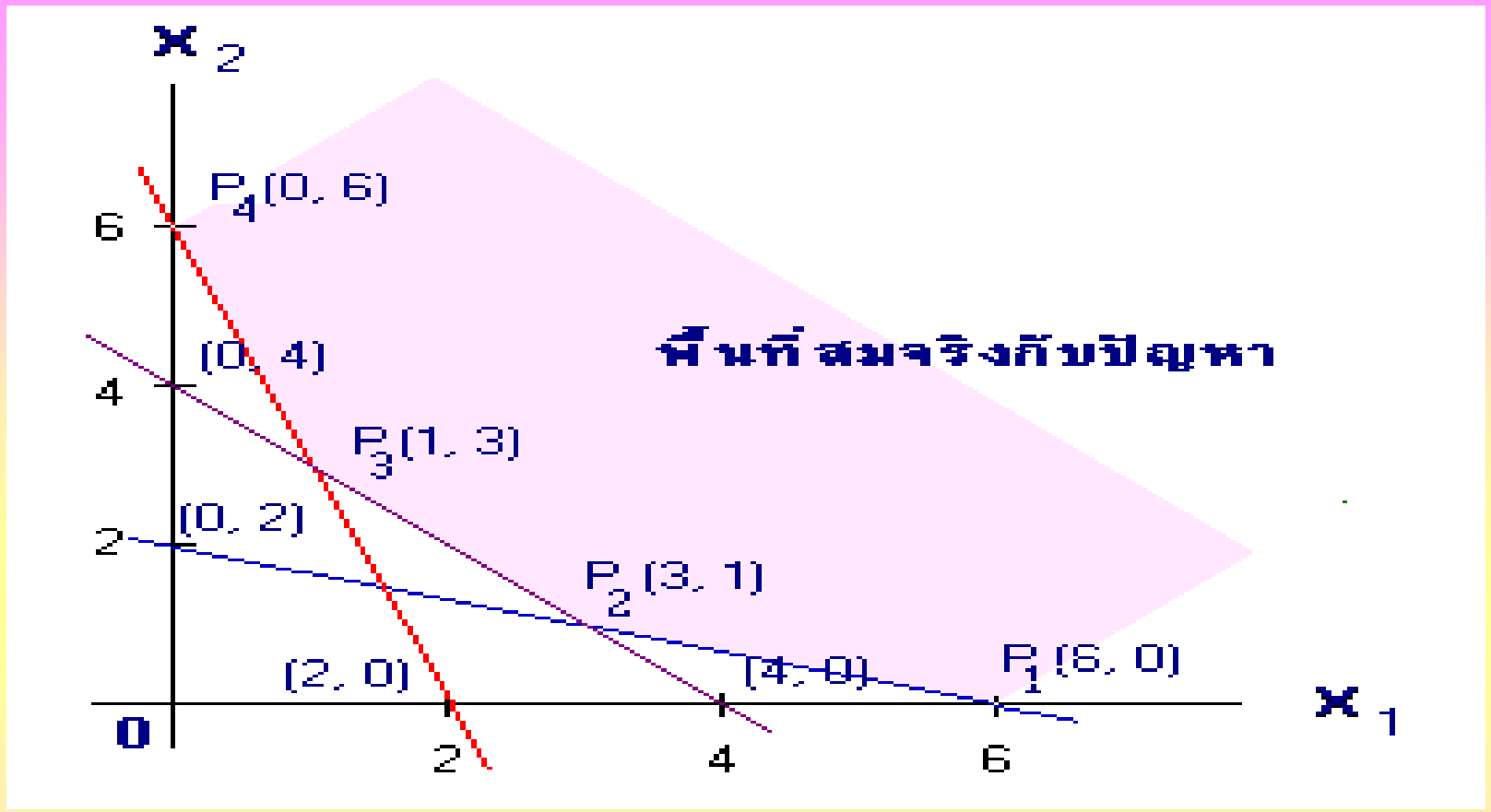
	P_1	P_2	P_3	P_4
X_1	6	3	1	0
X_2	0	1	3	6
Cost min	240,000	152,000	136,000	192,000

Minimum Cost = 40,000 × 1 + 32,000 × 3

= 136,000 baht/week

Mine A (x_1 = 1 days/week)

Mine B (x_2 = 3 days/week)



Minimum Cost = 40,000 × 1 + 32,000 × 3

= 136,000 baht/week

Ex.2 Maximize Profit Problem

	Type	Raw Mat(hrs)	Form Prep(hrs)	Quality Test(hrs)
X_1	1	6	3	4
X_2	2	6	6	2

$\frac{Pcs}{week}$	Condition		
	Raw Mat	420	hrs/week
	Form Prep	300	hrs/week
	Quality	240	hrs/week

$$6x_1 + 6x_2 \leq 420$$

$$3x_1 + 6x_2 \leq 300$$

$$4x_1 + 2x_2 \leq 240$$

$$\frac{x_1}{70} + \frac{x_2}{70} \leq 1$$

$$\frac{x_1}{100} + \frac{x_2}{50} \leq 1$$

$$\frac{x_1}{60} + \frac{x_2}{120} \leq 1$$

Target

Profit of Type 1 300 baht/piece

Profit of Type 2 200 baht/piece

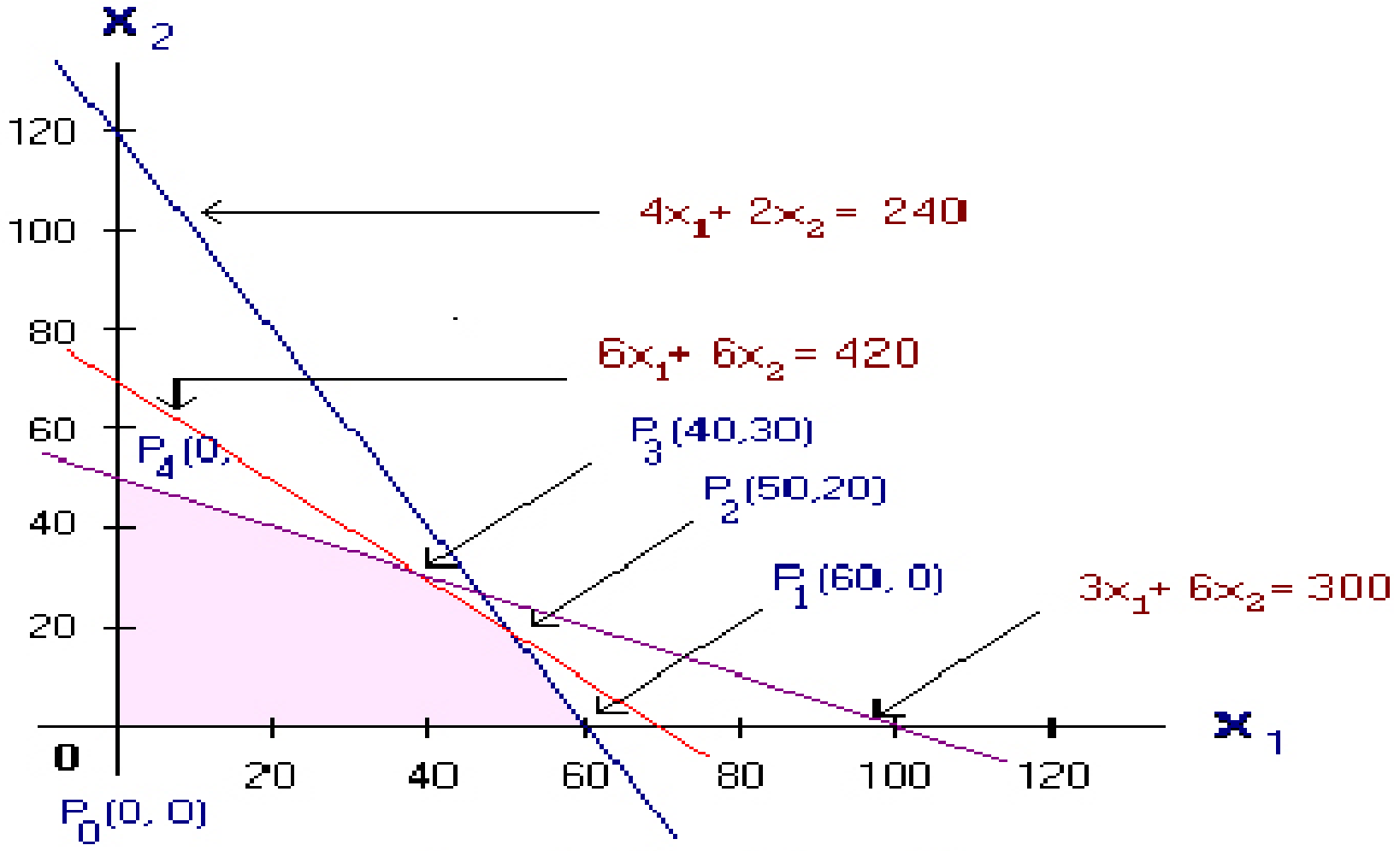
Target Equation

Profit = $300x_1 + 200x_2 = \max$

	P_1	P_2	P_3	P_4
X_1	60	50	40	0
X_2	0	20	30	50
Profit	18,000	19,000	18,000	10,000

Maximum Profit = $300 \times 50 + 200 \times 2$

= 19,000 baht/week



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Maximum Profit = 300 × 50 + 200 × 2

= 19,000 baht/week

Exam.#1

- Factory want to produce 2 items (A and B) by this ingredient

	w1	w2
A	4	4
B	2	6

Order w1 at least 360 units

Order w2 at least 240 units

If we want to get the minimum cost, how many A and B do we have to produce?

Cost of A 10,000 baht/piece

Cost of B 20,000 baht/piece

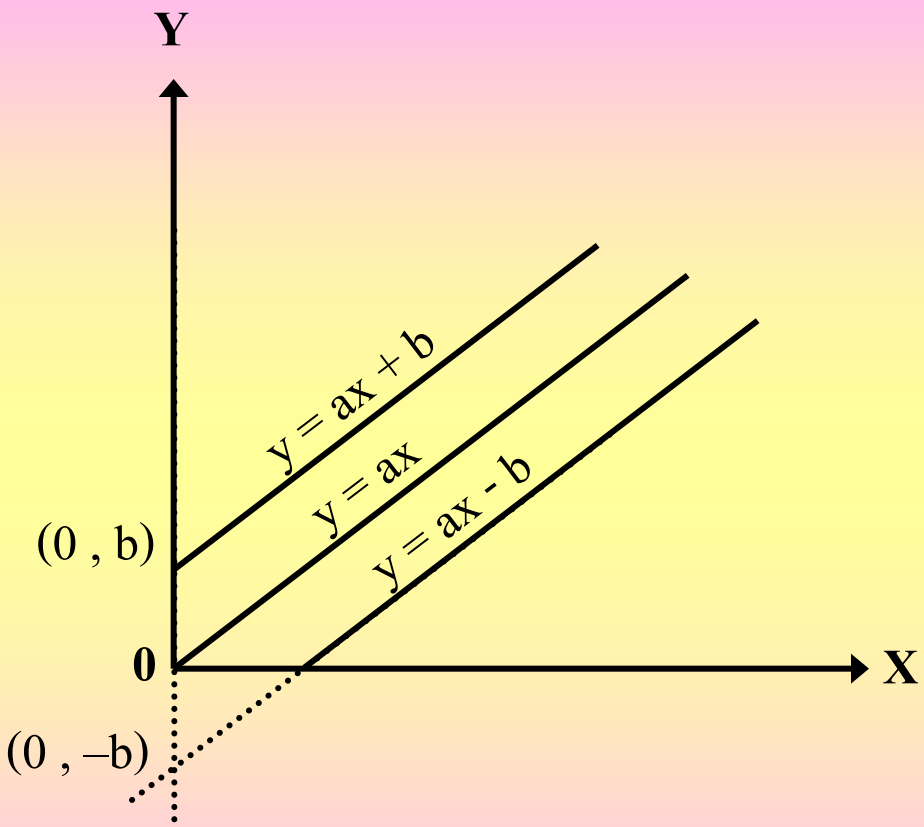
Exam.#2

- Factory want to produce 2 items (A and B) by this ingredient

	C	D
A	3	4
B	2	6

Supply capacity of C is 240 units
Supply capacity of D is 300 units
How many product that we get the maximum profit?
Profit of A 100 baht/unit
Profit of B 200 baht/unit

Linear model



$$y = ax + b$$

X = independent variable

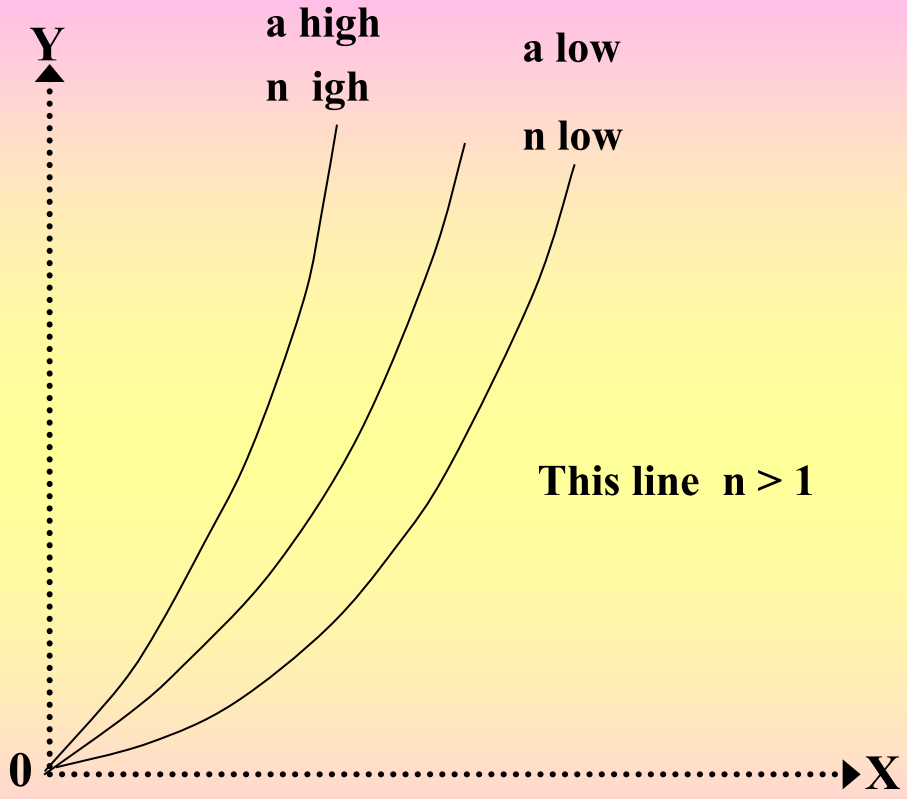
Y = dependent variable

a = slope

b = y-intercept

4.5 Non-linear Model

4.5.1 Power Model



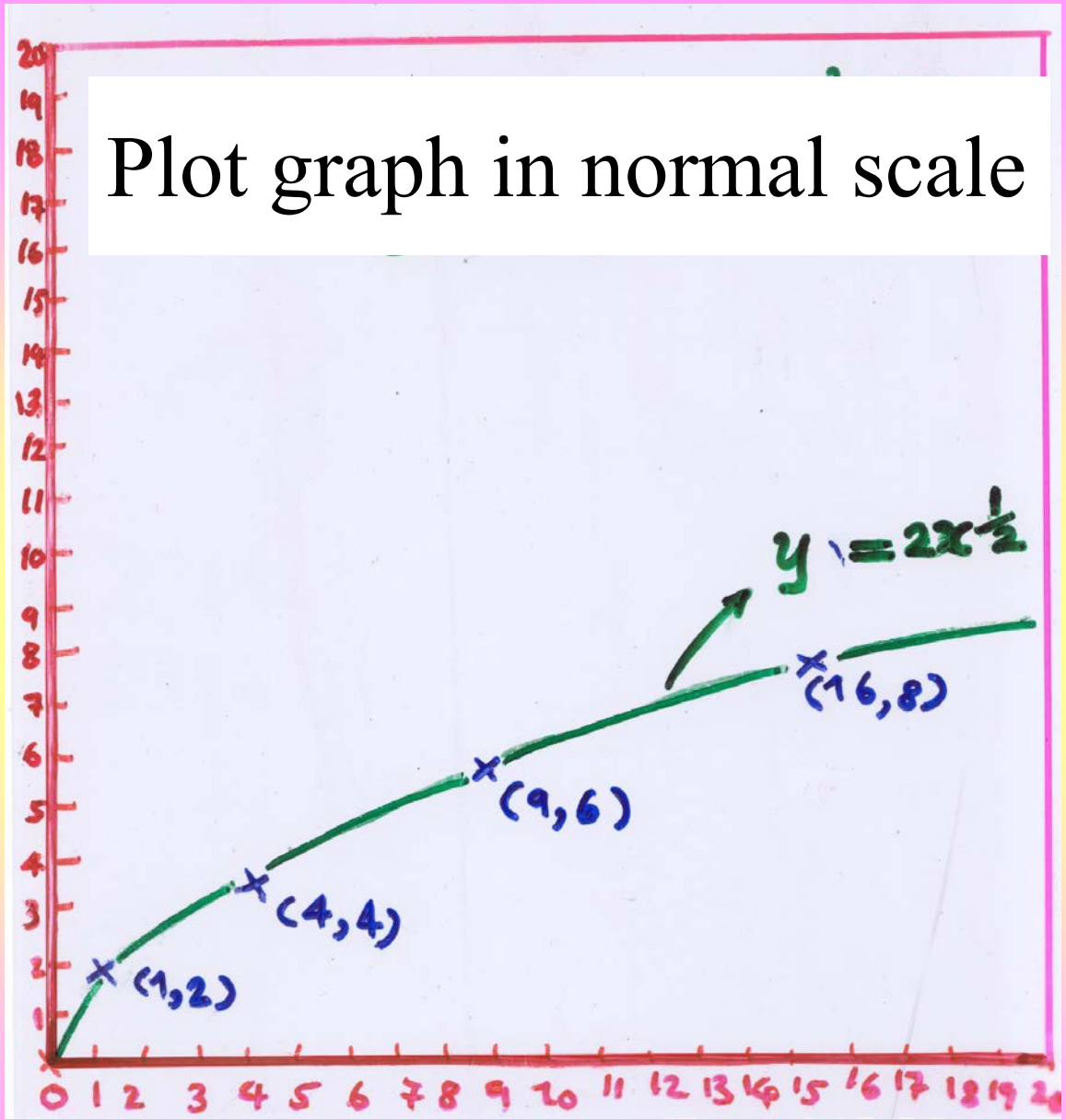
$$y = ax^n$$

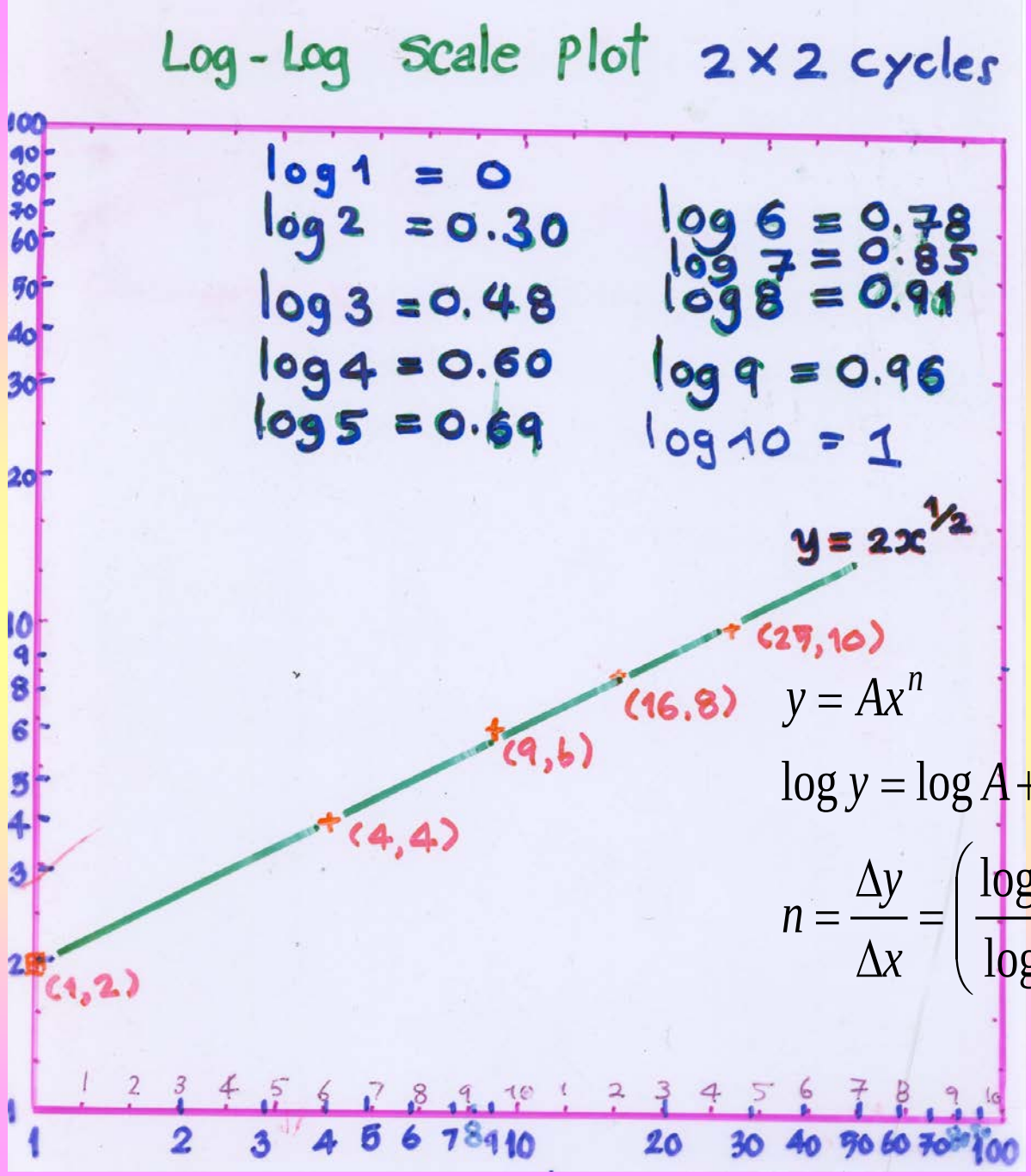
a = constant

n = power

X	1	4	9	16	25	36
Y	2	4	6	8	10	12

Plot graph in normal scale



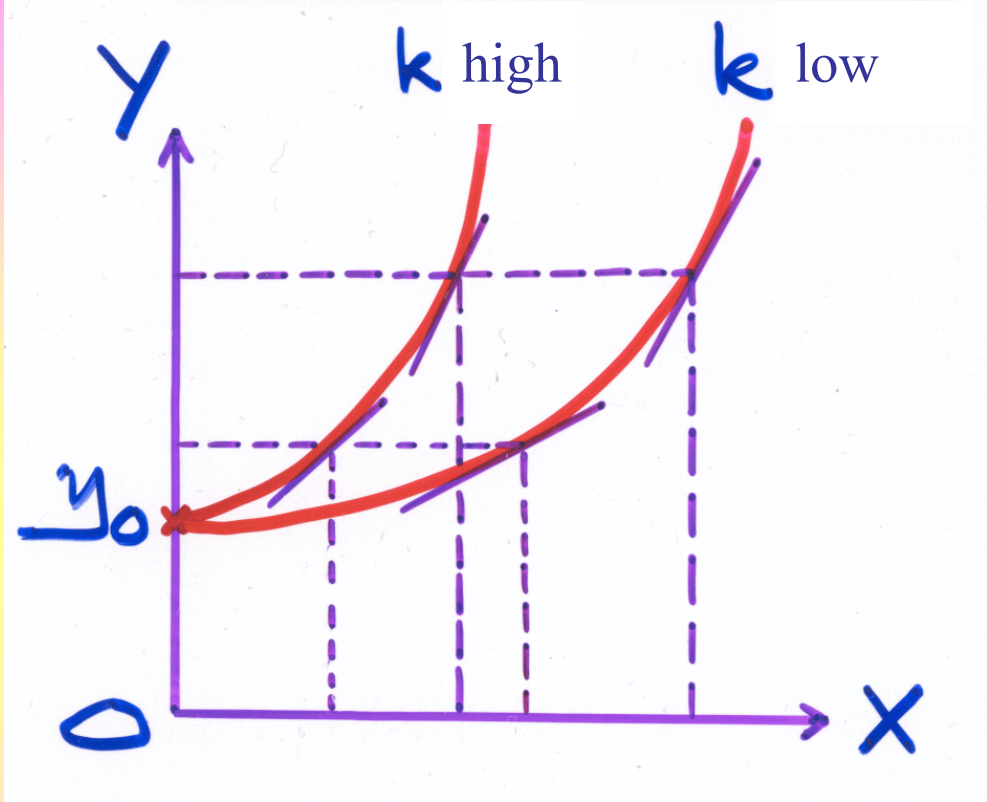


$$y = Ax^n$$

$$\log y = \log A + n \log x$$

$$n = \frac{\Delta y}{\Delta x} = \left(\frac{\log 4 - \log 2}{\log 4 - \log 1} \right) = \left(\frac{0.6 - 0.3}{0.6 - 0} \right) = \frac{1}{2}$$

4.5.2 Exponential model



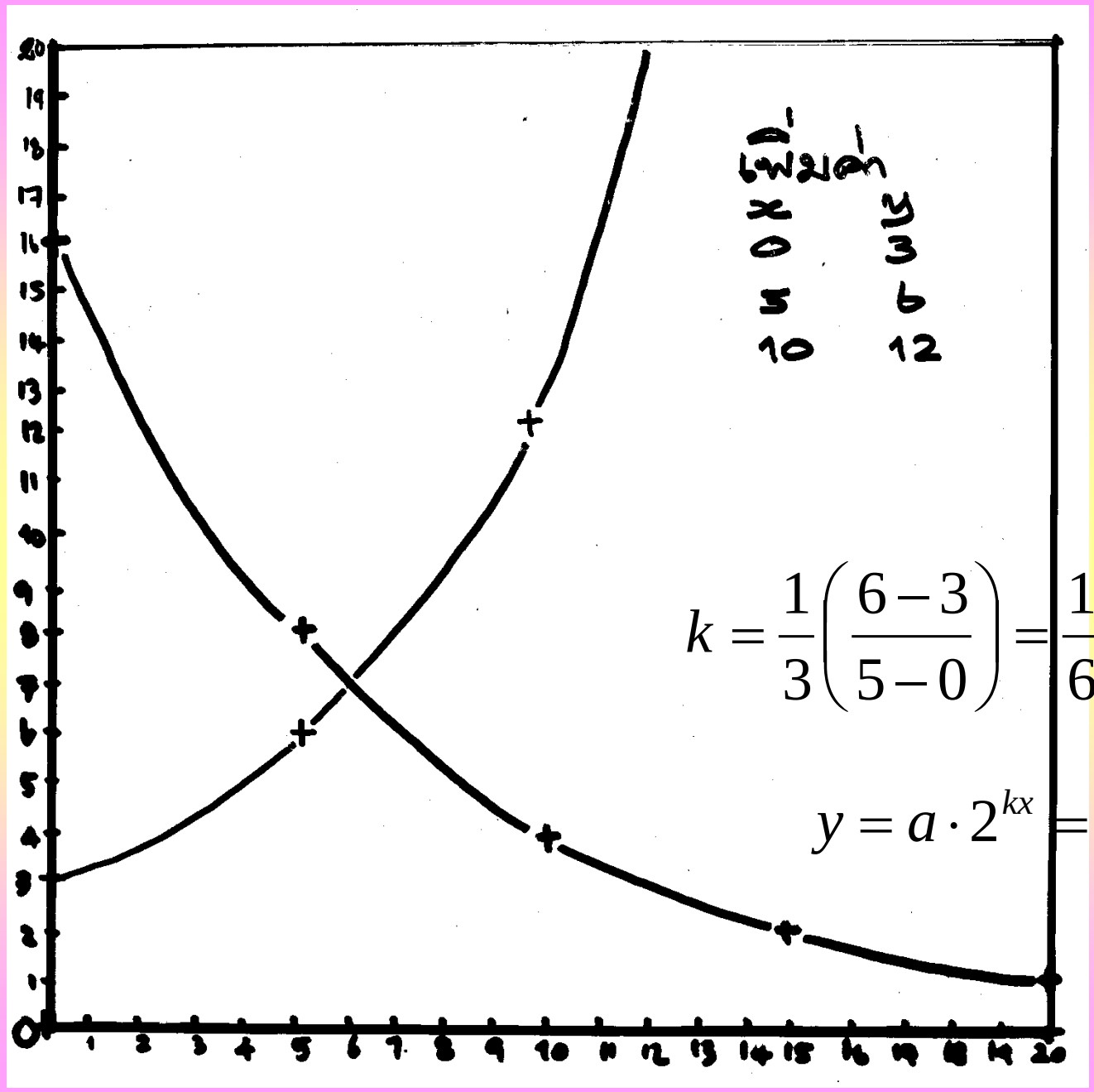
$$y = ae^{kx}$$

x=0, y=a

$$k = \frac{1}{y}\left(\frac{\Delta y}{\Delta x}\right)$$

e = if independent variable values are equally spaced,

the quotient of consecutive y-values is the same.



$$y = ae^{kx}$$

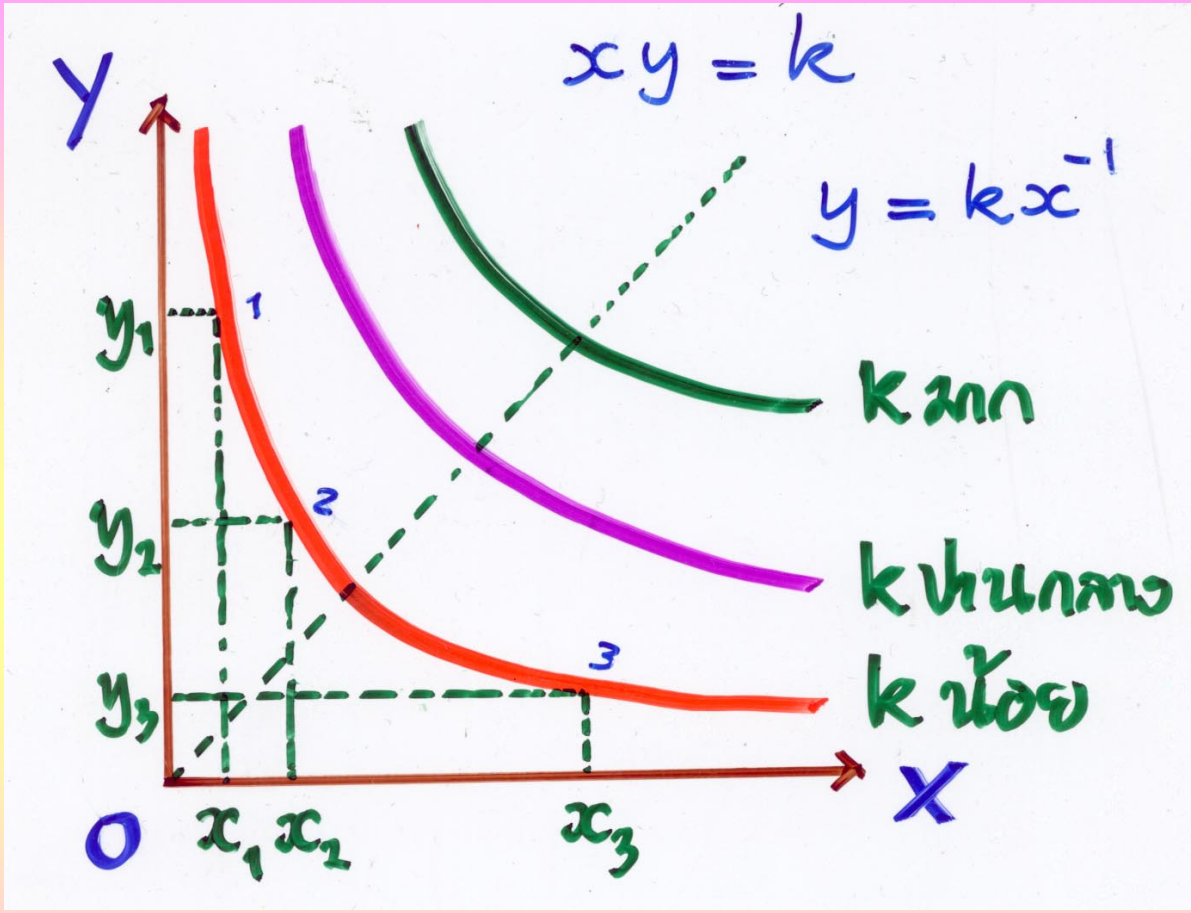
$x=0, y=a$

$$k = \frac{1}{y}\left(\frac{\Delta y}{\Delta x}\right)$$

$$k = \frac{1}{3}\left(\frac{6-3}{5-0}\right) = \frac{1}{6}\left(\frac{12-6}{10-5}\right) = \frac{1}{5}$$

$$y = a \cdot 2^{kx} = 3 \cdot 2^{x/5}$$

4.5.3 **Rectangular Hyperbolas**



x high y low

x moderate y moderate

x low y high

Multiple of x and y will be constant = k

Exam. 1. find function, plot graph, and solve c

X	1	8	27	.	.	c
Y	5	10	15	.	.	30

log 1 =0

log 2 =0.3

log 3= 0.48

log 4 =0.60

log 5 =0.69

log6 =0.78

log7 =0.85

log8 =0.90

log9 =0.96

log10 =1.00

log a x b = log a +log b

log a^n = n (log a)

2 find function, plot graph, and solve d

X	3	4	6	.	.	d
Y	36	27	18	.	.	2

3 find function, plot graph, and solve m

x	0	5	10	15	
y	16	8	4	2	0.25

3 find function, plot graph, and solve n

x	0	1	2	3	4	n
y	2.0	2.4	2.75	3.1	3.5	13.25

Questions?

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Reference:

1. วีระศักดิ์ อุดมโชค และคณะ, คณิตศาสตร์และคอมพิวเตอร์ในชีวิตประจำวัน,หนังสือ,2549
2. วีระศักดิ์ อุดมโชค , Lecture note คณิตศาสตร์และคอมพิวเตอร์ในชีวติประจำวัน,2551,
   ภาควิชาวิทยาศาสตร์พื้นภิภพ คณะวิทยาศาสตร์ มก.