

# Geometric Correction of Imagery

Present by:  
Dr. Weerakaset Suanpaga  
D.Eng(RS&GIS)

<http://pirun.ku.ac.th/~fengwks/rs/>

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## Geometric Correction of Imagery

- ➔ The intent is to compensate for the distortions introduced by a variety of factors, so that corrected imagery will have the geometric integrity of a planimetric map
- ➔ Systematic distortions: predictable; corrected by mathematical formulas. These often corrected during preprocessing
- ➔ Nonsystematic or random distortions: corrected statistically by comparing with ground control points. Often done by end-user

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## Geometric Correction

- ➔ Image includes geometric distortions
  - Internal distortions
  - External distortions
- ➔ Geometric correction is
  - To remove internal distortions
  - To remove external distortions
  - To identify the location of each pixels on the earth  
Overlay with map and other images
  - To convert coordinate system ( such as lat-lon -> UTM )
- ➔ Parameter determination
  - Systematic
  - Non-Systematic ( Black box type )
  - Combined

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## Geometric Correction at End Users

- ➔ High-resolution RS image is usually provided with UTM coordinate System after systematic geometric correction.
- ➔ However, the accuracy of the systematic geometric correction of usual product is not satisfactory. It is depending on position and attitude measurement of satellites. (Landsat : several hundred meters )
- ➔ Users must do geometric correction using GCP (Ground Control Point ) to overlay the image with Maps, GPS reading and etc. ( accuracy can be 1 pixel )

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## Distortions

- ➔ Geometric distortion is an error on an image, between the actual image coordinates and the ideal image coordinates which would be projected theoretically with an ideal sensor and under ideal conditions.
- ➔ Geometric distortions are classified into **internal distortion** resulting from the geometry of the sensor, and **external distortions** resulting from the attitude of the sensor or the shape of the object

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## Internal Distortions

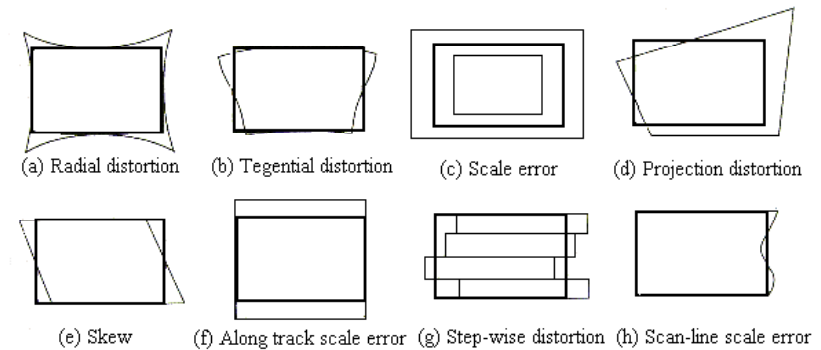
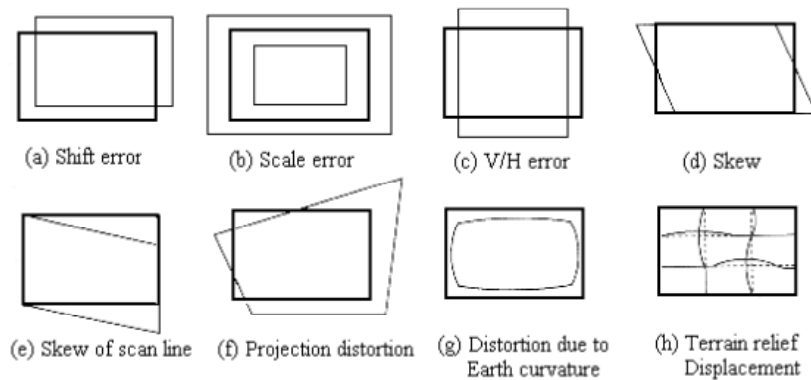


Figure 9.3.1 Internal Distortions

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## External Distortions



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Table 9.3.1 Causes of Geometric Distortions and their Types

Causes of distortions	Sensors			Slant Range Type <sup>4)</sup>
	Frame Sensor <sup>1)</sup>	Line Sensor <sup>2)</sup>	Point Sensor <sup>3)</sup>	
<b>a. Internal distortions</b>				
Radial distortion of lens	a	a	a	—
Tangential distortion of lens	b	b	b	—
Error of focal length	c	c	c	—
Tilt of projection plane	d	d	d	—
Non-flatness of projection plane	non-linear	non-linear	non-linear	—
Alignment error of array	—	e	g	—
Variation of sampling rate	—	f	h	f, h
Timing error of sampling	—	—	g	—
Variation of mirror velocity	—	—	h	—
<b>b. External distortions</b>				
Planimetric error of platform	a	a	a	a
Altitude error of platform	b	b	b	b
Motion of orbital position	—	c, d	c, d, e	c, d, e
Altitude of platform	f	f	f	f
Variation of attitude	—	f	f	f
Rotation of the Earth	—	d	d	d
Earth curvature	g	g	g	g
Terrain relief	h	h	h	h
Atmospheric refraction	non-linear	non-linear	non-linear	non-linear

<sup>1)</sup> Frame camera      <sup>2)</sup> SPOT HRV  
<sup>3)</sup> LANDSAT TM, MSS      <sup>4)</sup> SAR

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## Registration vs. Rectification

- Registration: reference one image to another of like geometry (I.e. same scale)
- Rectification: process by which the geometry of an image area is made planimetric by referencing to some standard map projection
- Reprojection: transformation from one projection system to another using a standard series of mathematical formulas

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## Rectification

- Whenever accurate area, direction and distance measurements are required, rectification is required.
- Integration of the image to a GIS: need to be able to georeference an individual's pixel location to its true location on the Earth's surface.
- Standard map projections: Lat/long, UTM, State Plane

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## Map Projections

- The manner in which the spherical (3d) surface of the Earth is represented on a flat (2-d) surface.
- Think about lighting up the interior of a transparent globe that has lines and features boundaries drawn on its surface. If the lighted globe is placed near a blank wall, you can see the shadow of the globe's map on the wall. You will have "projected" a 3d surface to a 2d surface.
- All projections involve some form of distortion

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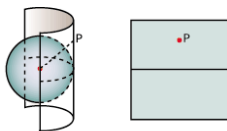
## Properties of Map Projections

- Conformality: preserves shape of small areas
- Equivalence: preserves areas, not shape
- Equidistance: scale of distance constant
- True direction: preserves directions, angles
- There is no single "perfect" map projection, all projections require some form of compromise

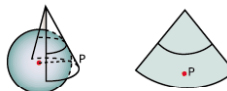
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## Families of Map Projections

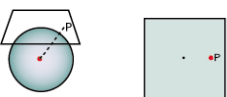
➔ Cylindrical



➔ Conical



➔ Azimuthal



Distortion is minimized along the line of contact

- Tangent: intersect along one line
- Secant: intersects along two lines

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Graphic from [www.pobonline.com](http://www.pobonline.com)

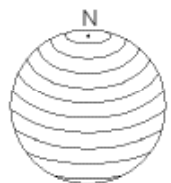
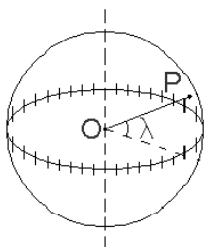
## Types of Coordinates

- ➔ Geographical: spherical coordinates based on the network of latitude and longitude lines, measured in degrees
- ➔ Planar: Cartesian coordinates defined by a row and column position on a planar (X,Y) grid, measured in meters or feet. Examples include: Universal Transverse Mercator (UTM) and State Plane

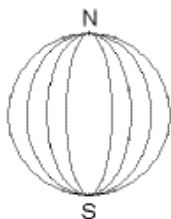
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## Latitude and Longitude Coordinates

Spherical coordinates are based on the network of latitude (N-S) and longitude lines (E-W), measured in angular degrees



Latitude lines or parallels



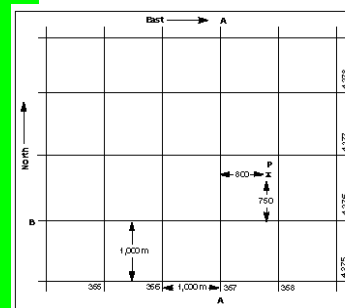
Longitude lines or meridians

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Graphics from: <http://www-istp.gsfc.nasa.gov/stargaze/Slatlong.htm>

## Universal Transverse Mercator (UTM)

UTM is a planar coordinate system



Referenced as Eastings & Northings, measured in meters



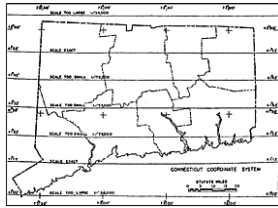
Globe subdivided into 60 zones - 6° longitude wide

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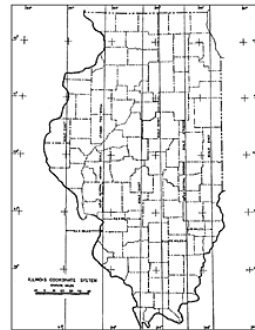
Graphics from: <http://mac.usgs.gov/mac/isb/pubs/factsheets/fs07701.html>

## State Plane Coordinate system

State Plane is a planar-rectangular coordinate system



Referenced as X & Y coordinates, measured in feet



Each state has its own coordinate system oriented to the particular shape of the state. SP based on a conformal map projection.

17 Graphics from : [www.pobonline.com/CDA/Article\\_Information/GPS\\_Index/1,2459,,00.html](http://www.pobonline.com/CDA/Article_Information/GPS_Index/1,2459,,00.html)

## Factors in Choosing a Map Projection

- Type of map
- Size and configuration of area to be mapped
- Location of map on globe: tropics, temperate or polar regions
- Special properties that must be preserved
- Types of data to be mapped
- Map accuracy
- Scale

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## Other Considerations

- Spheroid: the sphere isn't a true sphere. Most projections require the definition of a spheroid (or ellipsoid), which is a model of the earth's shape. Choose the spheroid appropriate for your geographic region.
- Datum: reference elevation, determined by National Geodetic Survey

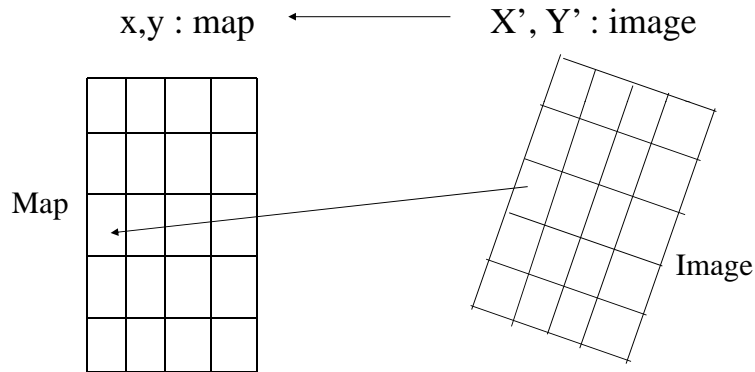
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## Coordinate Transformations

- Both registration and rectification require some form of coordinate transformation
- Transformation equation allows the reference coordinates for any data file location to be precisely estimated; to transform from one coordinate space to another

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## Transforming from one Coordinate System to Another



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## Transformation Equation

- To transform from the data file coordinates to a reference coordinate system requires a change in:
  - Scale
  - Rotation
  - Translation

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## Example: Transformation Equation

from Jensen 1996 p 133

$$X' = -382.08 + 0.034187x - 0.005481y$$

$$Y' = 130162 - 0.005576x - 0.0349150y$$

x,y : map                      X', Y' : image

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## Example: Transformation Equation

from Jensen 1996 p 133

$$X' = -382.08 + 0.034187x - 0.005481y$$

$$Y' = 130162 - 0.005576x - 0.0349150y$$

x,y : map                      X', Y' : image

Given an x, y location, compute the X', Y' location.

$$x = 598,285 \text{ E} \qquad y = 3,627,280 \text{ N}$$

$$X' = -382.08 + 20453.5 - 19881.1 = 190.3$$

$$Y' = 130162 - 3336.0 - 126646.5 = 180$$

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## Calculating Transformation Equations

- The statistical technique of least squares regression is generally used to determine the coefficients for the coordinate transformation equations.
- Want to minimize the residual error (RMS error) between the predicted ( $X'$ ) and observed ( $X'_{orig}$ ) locations
- $RMS\ error = \sqrt{(X' - X'_{orig})^2 + (Y' - Y'_{orig})^2}$

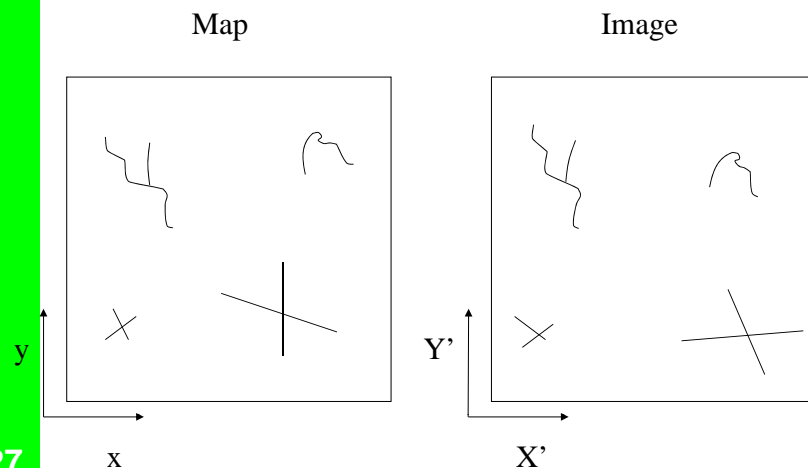
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## Ground Control Points (GCP)

- Ground control points (GCPs): locations that are found both in the image and in the map
- The GCP coordinates are recorded from the map and from the image and then used in the least squares regression analysis to determine the coordinate transformation equations
- Want to use features that are readily identifiable and not likely to move, e.g., road intersections

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## Ground Control Points (GCP)



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## 1st order transformation

- Also referred to as an affine transformation
- $X' = a_0 + a_1x + a_2y$   $X', Y' = \text{image}$   
 $Y' = b_0 + b_1x + b_2y$   $x, y = \text{map}$
- $a_0, b_0$ : for translation
- $a_1, b_1$ : for rotation and scaling in x direction
- $a_2, b_2$ : for rotation and scaling in y direction
- 6 unknowns, need minimum of 3 GCP's
- difference in scale in X & Y, true shape not maintained but parallel lines remain parallel

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## 2nd order polynomial

- ➔  $X' = a_0 + a_1x + a_2y + a_3x^2 + a_4y^2 + a_5xy$   
 $Y' = b_0 + b_1x + b_2y + b_3x^2 + b_4y^2 + b_5xy$
- ➔ 12 unknowns, need minimum of 6 GCP's
- ➔ nonlinear differences in scale in X & Y, true shape not maintained, parallel lines no longer remain parallel
- ➔ Higher order polynomials also known as rubber sheeting

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## Minimum of GCP's

Higher orders of transformation can be used to correct more complicated types of distortion. However, to use a higher order of transformation, more GCP's are needed. The minimum number of GCP's required to perform a transformation of order  $t$  equals:

$$\frac{((t+1)(t+2))}{2}$$

$t$ Order	Min. # of GCP's
1	3
2	6
3	10
4	15

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## Ortho-rectification

- ➔ Orthorectification: correction of the image for topographic distortion. This is especially important in mountainous areas where there can be distortions (e.g. scale changes) due to the varying elevations in the image scene.
- ➔ Corrected by including Z values (elevation from a DEM) and photogrammetric principles

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## Steps of Usual Non-systematic Geometric Correction

- ➔ **Locate the image exactly on a map coordinate, by re-projecting the image, so that location of each pixel can be identified correctly**
1. Make a list of Image Coordinate and their position using GCP.
  2. GCP ( Ground Control Point ) is taken from such as intersection, corner of reclamations, capes, small islands, river joint point and etc.
  3. GCP should be able to be identified both in image and map ( or GPS )
  4. If Map is not available, GPS is used to obtain coordinates.
  5. Select an appropriate transformation equation ( usually affine is OK )
  6. Re-project the image into the map coordinate
  7. usually Cubic Convolution is used as Re-sampling method.



Image Coordinate		Map Coordinate	
Column	Line	Easting	Northing
390	400	524,445	1,234,456
700	420	525,445	1,234,256
...	...	...	...
...	...	...	...
...	...	...	...
...	...	...	...
...	...	...	...
...	...	...	...

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## Geometric Correction

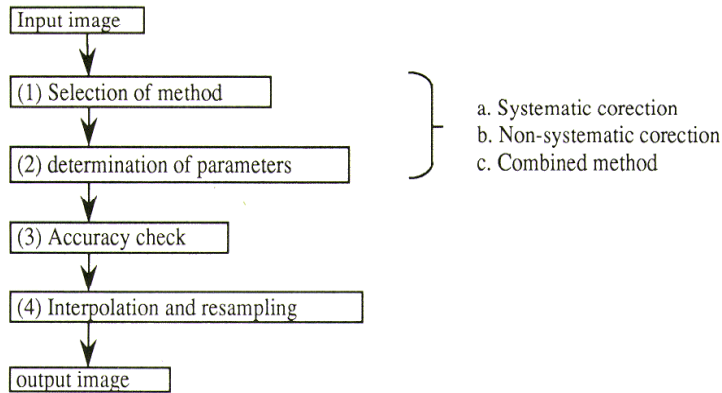


Figure 9.4.1 The flow of geometric correction

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Following types of points are often chosen for GCP

- boundaries of land and water, e.g.
- corner of reclamations, dams, breakwaters,
- corner of harbors,
- capes, lighthouses, small islands,
- factories, buildings, runways of airport,
- center of small ponds,
- river joint point, road cross point

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## Rectification Process-conclusion

- Locate ground control points
- Compute and test a coordinate transformation matrix. Iterative process till required RMS error tolerance is reached.
- Create an output image in the new coordinate system. The pixels must be resampled to conform to the new grid.

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## Root Mean Square (RMS) Error

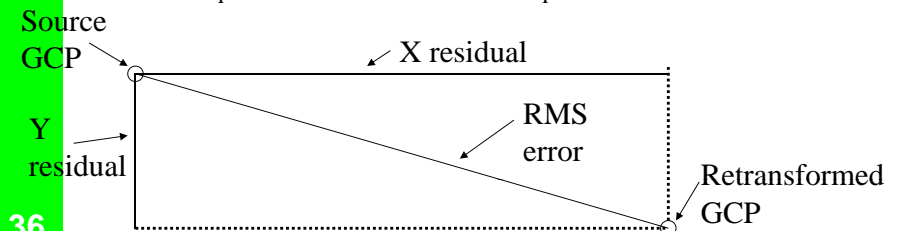
RMS Error per GCP,  $R_i$ :

$$R_i = \text{SQRT}(XR_i^2 + YR_i^2)$$

where  $R_i$  = RMS Error per GCP<sub>*i*</sub>

$XR_i$  = X residual for GCP<sub>*i*</sub>

$YR_i$  = Y residual for GCP<sub>*i*</sub>



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## Total RMS Error

X RMS error,  $R_x$

Y RMS error,  $R_y$

$$R_x = \text{SQRT}(1/n (\text{SUM } XR_i^2)) \quad R_y = \text{SQRT}(1/n (\text{SUM } YR_i^2))$$

where  $XR_i$  - X residual for GCP<sub>i</sub>

$YR_i$  = Y residual for GCP<sub>i</sub>

$n$  = # of GCPs

$$\text{Total RMS, } T = \text{SQRT}(R_x^2 + R_y^2)$$

Error Contribution by Individual GCP,  $E_i$  :

$$E_i = R_i / T$$

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## RMS Error: Example

X				Y			
$x_i$	$x_r$	Resid	Resid <sup>2</sup>	$y_i$	$y_r$	Resid	Resid <sup>2</sup>
228.0	228.5	0.5	0.25	242.0	241.3	-0.7	0.49
152.0	151.5	-0.5	0.25	237.0	237.0	0.0	0.0
199.0	199.2	0.2	0.04	318.0	317.4	-0.6	0.36
174.0	173.8	-0.2	0.04	352.0	352.1	0.1	0.01
42.0	41.9	-0.1	0.01	430.0	435.5	0.5	0.25
TotalSum			0.59				1.11

$$\text{X RMS Error} = \text{RMS}_x = \text{SQRT}(0.59/5) = \text{SQRT}(0.118) = \mathbf{0.34}$$

$$\text{Y RMS Error} = \text{RMS}_y = \text{SQRT}(1.11/5) = \text{SQRT}(0.222) = \mathbf{0.47}$$

$$\text{Total RMS Error} = \text{SQRT}(0.118 + 0.222) = \text{SQRT}(0.340) = \mathbf{0.58}$$

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## RMS Error: Example

### Error Contribution per GCP

	$R_x$	$R_y$	$R_i$	$E_i$
GCP1	0.25	0.49	0.86	1.48
GCP2	0.25	0.0	0.50	0.86
GCP3	0.04	0.36	0.64	1.10
GCP4	0.04	0.01	0.22	0.38
GCP5	0.01	0.25	0.51	0.88

where  $R_i$  = RMS Error per GCP

$E_i$  = Error Contribution by Individual GCP<sub>i</sub>

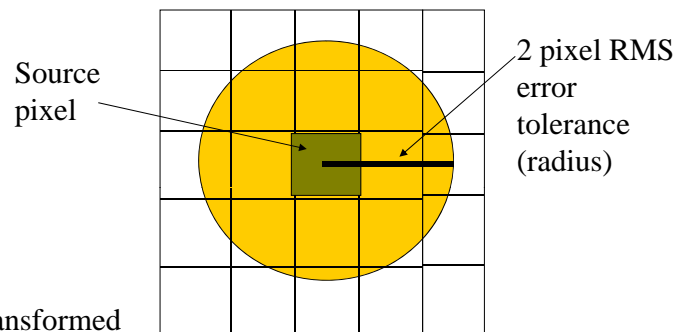
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## RMS Error Tolerance

- Rectification always entails some amount of error. The amount of error that the user is willing to tolerate depends on the application.
- RMS error is thought of as a error tolerance radius or window. The re-transformed pixel is within the window formed by the source pixel and a radius buffer
- For example: a RMS of 0.5 pixel would still put the transformed coordinate within the source pixel

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## RMS Error Tolerance

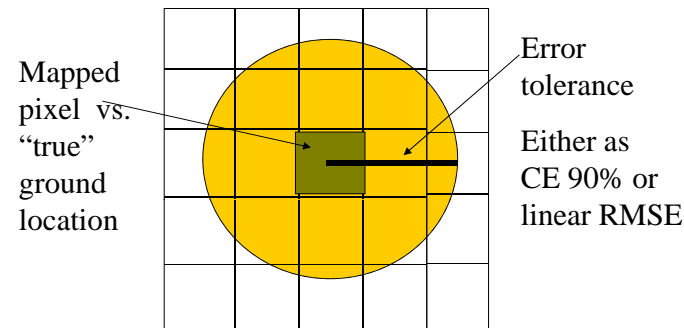


Retransformed coordinates within this range are considered correct

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From ERDAS Imagine Field Guide 5th ed.

## Geometric Accuracy: how close do you need?



The greater the geometric accuracy the higher the cost

Scale	CE90%	RMSE
1:50,000	25.5m	15.4m
1:5,000	4.2m	2.6M

[http://www.digitalglobe.com/product/ortho\\_imagery.shtml](http://www.digitalglobe.com/product/ortho_imagery.shtml)

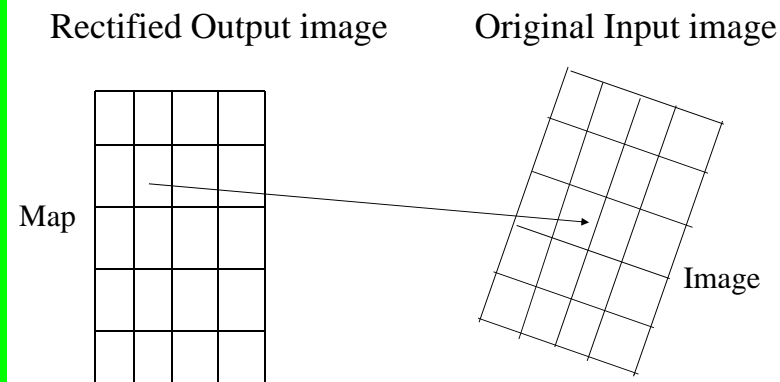
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## Resampling process

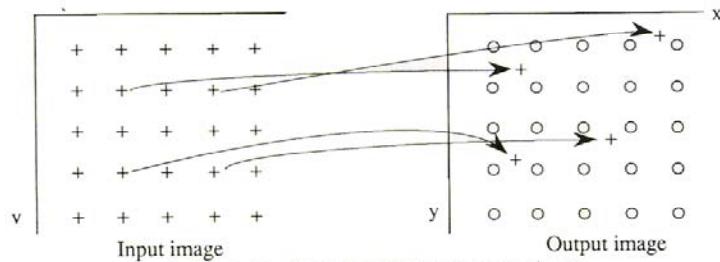
- The rectification/resampling process at first glance appears to operate backwards in that the transformation equation goes from the map to the image, when you might expect it to go from the image to the map.
- The process works as follows:
  - create output grid
  - backtransform from map to image
  - resample to fill output grid

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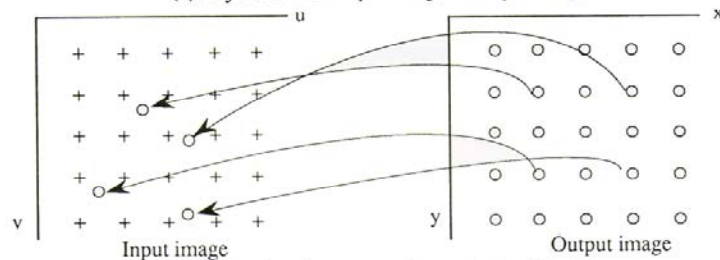
## Resampling: for each output pixel calculate the closest input coordinate



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(a) Projection from input image to output image



(b) Projection from output image to input image

Figure 9.7.1 Resampling Method

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## Resampling process

- An empty output matrix is created in the map coordinate systems at the desired starting coordinate location and pixel size
- Using the transformation equation, the coordinates of each pixel in the output map matrix are transformed to determine their corresponding location in the original input image
- This transformed cell location will not directly overlay a pixel in the input matrix. Resampling is used to assign a DN value to the output matrix pixel - determined on the basis of the pixel values which surround its transformed position in the original input image matrix

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## Resampling Methods

### (1) Nearest neighbor (NN)

- The nearest point will be sampled. The geometric error will be a half pixel at maximum. It has the advantage of being easy and fast. The data value will not change.

### (2) Bi-linear (BL)

- The bi-linear function is applied to the surrounding four points. The spectral data will be smoothed after the interpolation.

### (3) Cubic convolution (CC)

- The spectral data will be interpolated by a cubic function using the surrounding sixteen points. The cubic convolution results in sharpening as well as smoothing, though the computation takes a longer time when compared with the other methods. Interpolation

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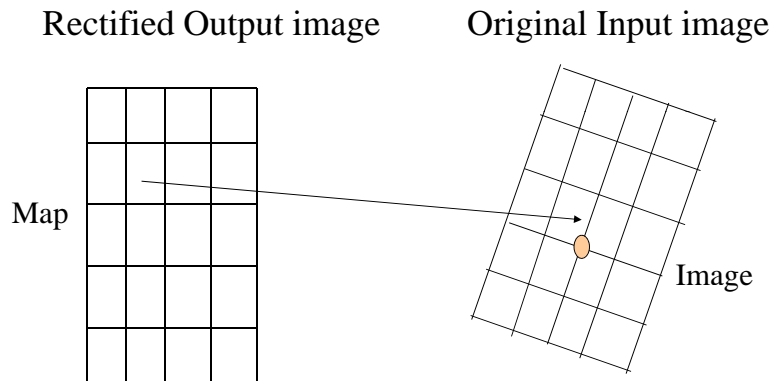
## Nearest-neighbor

- Substitutes in DN value of closest pixel. Transfers original pixel brightness values without averaging them. Keeps extremes.
- Suitable for use before classification
- Fastest to compute
- Results in blocky "stair stepped" diagonal lines

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## Nearest neighbor resampling

: chose closest single pixel



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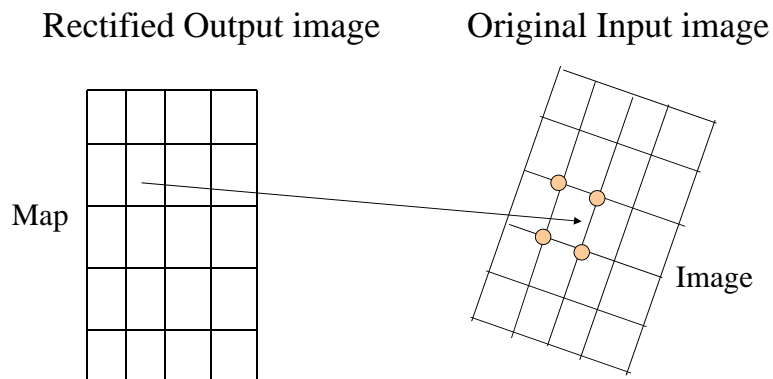
## Bilinear Interpolation

- Distance weighted average of the DN's of the four closest pixels
- More spatially accurate than nearest neighbor; smoother, less blocky
- Still fast to compute
- Pixel DN's are averaged, smoothing edges and extreme values

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## Bilinear Interpolation

: weighted average of four closest pixels



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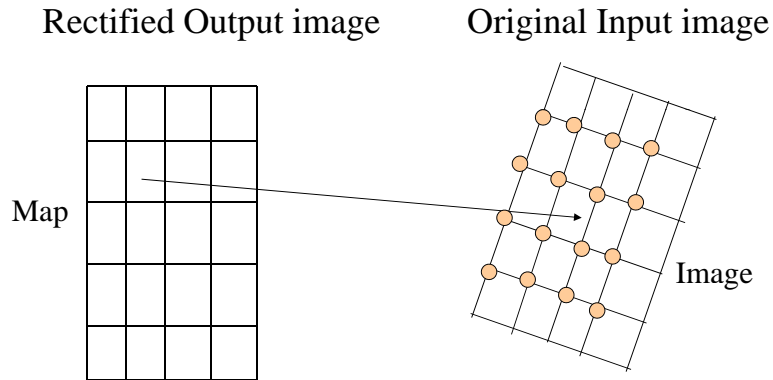
## Cubic Convolution

- Evaluates block of 16 nearest pixels, not strictly linear interpolation
- Mean and variance of output distribution match input distribution, however data values may be altered
- Can both sharpen image and smooth out noise
- Computationally intensive

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# Cubic Convolution

: weighted average of 16 closest pixels



(a) Nearest neighbor:  $Q(u,v) = P_{k,l}$  where  $k = [u+0.5]$  and  $l = [v+0.5]$ .

(b) Bi-linear:  $Q(u,v) = (1-s)(1-t)P_{i,j} - (1-s)t P_{i,j+1} + s(1-t) P_{i+1,j} + s t P_{i+1,j+1}$

(c) Cubic convolution:  $Q(u,v) = [f(1+t) \ f(t) \ f(1-t) \ f(2-t)] \begin{bmatrix} P_{1,1} & P_{1,2} & P_{1,3} & P_{1,4} \\ P_{2,1} & P_{2,2} & P_{2,3} & P_{2,4} \\ P_{3,1} & P_{3,2} & P_{3,3} & P_{3,4} \\ P_{4,1} & P_{4,2} & P_{4,3} & P_{4,4} \end{bmatrix} \begin{bmatrix} f(1+s) \\ f(s) \\ f(1-s) \\ f(2-s) \end{bmatrix}$

Let  $f(x) = \sin(\pi x) / (\pi x)$   
 $\approx \begin{cases} 1 - 2|x|^2 + |x|^3 & (0 \leq |x| < 1) \\ 4 - 8|x| + 5|x|^2 - |x|^3 & (1 \leq |x| < 2) \\ 0 & (2 \leq |x|) \end{cases}$

then  $f(1+y) = -y(1-y)^2$      $f(y) = (1-y) + y(1-y)^2$   
 $f(1-y) = y + y^2(1-y)$      $f(2-y) = -y^2(1-y)$

$Q(u,v)$  : A point to be interpolated  
 $P_{i,j}$  : Input image data  
 $[ \ ]$  : Gaussian symbol to make integer  
 $s = u - [u]$ ,  $t = v - [v]$

Figure 9.7.2 Interpolation Method

# Convolution

➔ N.N Nearest Neighbor

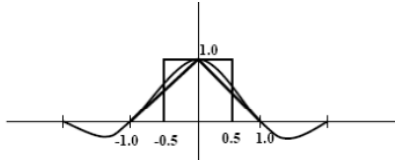
$$h(x) = \begin{cases} 1 & |x| \leq \frac{1}{2} \\ 0 & \frac{1}{2} < |x| \end{cases}$$

➔ B.L. Bi-Linear

$$h(x) = \begin{cases} 1 - |x| & |x| \leq 1 \\ 0 & 1 < |x| \end{cases}$$

➔ C.C Cubic Convolution

$$h(x) = \begin{cases} 1 - 2|x|^2 + |x|^3 & |x| \leq 1 \\ 4 - 8|x| + 5|x|^2 - |x|^3 & 1 \leq |x| \leq 2 \\ 0 & 2 \leq |x| \end{cases}$$



# END

Thank you for  
Attention