Notes on ICNS 100

Chapter 2: 2.5 Graphs in Rectangular Coordinates

1. An <u>x-intercept</u> of the graph of an equation in x and y is a point where the graph intersects the x-axis. A <u>y-intercept</u> is a point where the graph intersects the y-axis.

   <u>Examples</u>

   (a) Find the x- and y-intercepts of the graph of $y = 2x + 3$, and sketch the graph.

   (b) Determine the intercepts, if any, of the graph of $s = 100/t$, and sketch the graph.

   (c) Determine the intercepts of the graph of $x = 3$, and sketch the graph.

2. Each function f gives rise to an equation, namely $y = f(x)$. Its graph consists of all points $(x, f(x))$, where x is in the domain of f.

   (a) Graph $f(x) = \sqrt{x}$. (Square-root function)

   (b) Graph $p = G(q) = |q|$. (Absolute-value function)

3. A <u>zero</u> of a function f is any value of x such that $f(x) = 0$.

4. The zeros of a function are precisely the x-intercepts of its graph.

5. How to determine the domain and range of a function by looking at its graph?

   Answer: In general, the domain consists of all x-values that are included in the graph, and the range is all y-values that are included.

   For example, the domain and range of $f(x) = \sqrt{x}$ are all nonnegative numbers, while the domain of $p = G(q) = |q|$ is all real numbers and the range is all $p \geq 0$.

   <u>Example</u>

   (a) Graph the case-defined function

$$f(x) = \begin{cases} x & \text{if } 0 \leq x < 3 \\ x - 1 & \text{if } 3 \leq x \leq 5 \\ 4 & \text{if } 5 < x \leq 7 \end{cases}$$

6. How to tell whether a curve is the graph of a function?

   Answer: Apply the <u>vertical-line test</u>: If a <u>vertical</u> line L can be drawn that intersects a curve in at least two points, then the curve is <u>not</u> the graph of a function of x, and when no such vertical line can be drawn, the curve <u>is</u> the graph of a function of x.

   <u>Example</u>

   (a) Graph $x = 2y^2$.

7. After we know that a curve in question is the graph of a function, how to determine whether this function is one-to-one?

   Answer: Apply the <u>horrizonal-line test</u>: If a <u>horrizontal</u> line L can be drawn that intersects the graph of a function in at least two points, then the function is <u>not</u> one-to-one, and when no such horrizontal line can be drawn, the function is one-to-one.

<u>Assignment</u> Do Problems 2.5: 6, 10, 12, 14, 30, 32, 36, 40.