Notes on ICNS 100

Chapter 2: 2.6 Symmetry

1. A graph is symmetric about the y-axis if and only if 
   \((-a,b)\) lies on the graph when \((a,b)\) does.

2. In practice, to test the symmetry about the y-axis, replace \(x\) by \(-x\) in the given equation and see if an equivalent equation is obtained. For example, the graph of \(y = x^2\) is symmetric about the y-axis.

3. A graph is symmetric about the x-axis if and only if \((a,-b)\) lies on the graph when \((a,b)\) does.

4. In practice, to test the symmetry about the x-axis, replace \(y\) by \(-y\) in the given equation and see if an equivalent equation is obtained. For example, the graph of \(x = y^2\) is symmetric about the x-axis.

5. We note that the constant function \(f(x) = 0, \) for all \(x,\) is symmetric about the x-axis. It turns out that this is the only *function* whose graph is symmetric about the x-axis.

6. A graph is symmetric about the origin if and only if \((-a,-b)\) lies on the graph when \((a,b)\) does.

7. In practice, to test the symmetry about the origin, replace \(x\) by \(-x\) and \(y\) by \(-y\) in the given equation and see if an equivalent equation is obtained. For example, the graph of \(y = x^3\) is symmetric about the origin.

Example

Graph the following equations using intercepts and symmetry:

(a) \(y = 1/x\)
(b) \(y = 1 - x^4\)
(c) \(4x^2 + 9y^2 = 36\)

8. A graph is symmetric about the line \(y = x\) if and only if \((b,a)\) lies on the graph when \((a,b)\) does.

9. In practice, to test the symmetry about the line \(y = x,\) we interchange the roles of \(x\) and \(y\) in the given equation and see if an equivalent equation is obtained. For example, the graph of \(x^2 + y^2 = 1\) is symmetric about the line \(y = x.\)

Example

Sketch the graph of \(g(x) = 2x + 1\) and its inverse in the same plane.

Assignment Do Problems 2.6: 8, 16, 20, 24, 27.