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Notes on ICNS 100

Chapter 4: 4.1 Exponential Functions

1. The function f defined by $f(x) = b^x$ where $b > 0$, $b \neq 1$, and the exponent x is any real number, is called an exponential function with base b .

2. Rules for Exponents

$$(a) a^m a^n = a^{m+n} \quad (e) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(b) \frac{a^m}{a^n} = a^{m-n} \quad (f) a^1 = a$$

$$(c) (a^m)^n = a^{mn} \quad (g) a^0 = 1$$

$$(d) (ab)^n = a^n b^n \quad (h) a^{-n} = \frac{1}{a^n}$$

3. Properties of the Exponential Function $f(x) = b^x$

(a) The domain of an exponential function consists of all real numbers.
The range consists of all positive numbers.

(b) The graph of $f(x) = b^x$ has y -intercept $(0, 1)$.
There is no x -intercept.

(c) If $b > 1$, the graph rises from left to right.
If $0 < b < 1$, the graph falls from left to right.

(d) If $b > 1$, the graph approaches the x -axis as x becomes more and more negative.
If $0 < b < 1$, the graph approaches the x -axis as x becomes more and more positive.

4. Compound Amount and Compound Interest

The compound amount S of a principal P at the end of n interest periods at the periodic rate of r is given by

$$S = P(1 + r)^n.$$

5. The Number e

The smallest real number that is greater than all of the numbers $\left(\frac{n+1}{n}\right)^n$ is denoted by the letter e in honor of the Swiss mathematician

Leonhard Euler (1707–1783). The number e is irrational (its decimal is nonrepeating). Its approximation up to 12 decimal places is

$$e \approx 2.718281828459.$$

Example graph each function using only transformation of the base graph $y = f(x) = b^x$.

(a) $y = f(x) = \left(\frac{1}{3}\right)^x$

(b) $y = f(x) = 2^{(x-1)^2}$

(c) $y = f(x) = 1 - 3^{-x-1}$

(d) $y = f(x) = \frac{1}{2}(2^{x/2})$

Example Suppose \$900 is placed in a saving account that earns interest at the rate of 4.5% compounded semiannually. (a) What is the value of the account at the end of five years? (b) If the account had earned interest at the rate of 4.5% compounded annually, what would be the value after five years?

Example A certificate of deposit is purchased for \$6500 and is held for six years. If the certificate earns 4% compounded quarterly, what is it worth at the end of six years?

Assignment Do Problems 4.1: 2, 4, 10, 24, 30.