ICNS100 Midterm, T4/2013-14

1. (a) The graph of \( y = f(x) \) is shown below.

Use the translation and/or reflection techniques to sketch the graphs of

i. \( y = f(x - 1) - 2 \) (Sketch the graph in the above coordinate plane. Label it by (a).) (2 points)

ii. \( y = f(-x) \) (Sketch the graph in the above coordinate plane. Label it by (b).) (1 point)

iii. \( y = 2 - f(x) \) (Sketch the graph in the above coordinate plane. Label it by (c).) (2 points)

(b) Test whether the graph of \( y = 5x - x^3 \) is symmetric about

i. the \( y \)-axis, (2 points)

ii. the origin, (2 points)

(c) If \( f \) is a linear function such that \( f(-1) = 2 \) and \( y \)-intercept is 1/4, find \( f(x) \). (3 points)
2. (a) Define a function \( f \) by \( f(x) = \begin{cases} 
3 + x, & \text{if } -4 \leq x < 1 \\
3 - x, & \text{if } 1 \leq x < 4.
\end{cases} \)

i. Sketch the graph of \( f(x) \). (3 points)

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ii. Find the domain of \( f(x) \). (1 point)

iii. Find the range of \( f(x) \). (1 point)

(b) Find a general linear equation of the line that passes through the points (3,11) and (1,-2). (3 points)

(c) Determine whether lines \( A \) and \( B \) below are parallel, perpendicular or neither. Justify your answer. (3 points)

Line \( A : 2x - 5y = -7 \)

Line \( B : 10x + 4y = 9 \).

(d) Find an equation of the line which is perpendicular to the line \(-x + 4y = 6\) and passes through the point (6,1). (3 points)

3. (a) Suppose that the cost of producing 10 pairs of shoes is 4000 baht and the cost of 20 pairs is 7000 baht. If the cost is linearly related to the number of shoes produced,

i. find the cost function in terms of the quantity, (3 points)

ii. find the cost for producing 35 pairs of shoes. (1 point)

(b) The demand function for a manufacturer’s product is \( p = 180 - 2q \), where \( p \) is the price (in baht) per unit when \( q \) units are demanded (per day) by consumers. Find the level of production that maximizes the manufacturer’s total revenue and determine this revenue. (4 points)
(c) Let \( f(x) = -x^2 + 4x + 1 \).

i. Sketch the graph of \( f(x) \). (4 points)

\[
\begin{array}{c|c|c|c|c|c}
  x & -4 & -3 & -2 & -1 & 0 \\
  y & & & & & \\
\end{array}
\]

ii. Find the range of \( f(x) \). (1 point)

iii. Find the equation of axis of symmetry. (1 point)

4. (a) Solve the following system. Write your answer in a parametric form if necessary.
\[
\begin{align*}
  x + 2y + z &= 4 \\
  3x + z &= 2 \\
  x - y + z &= 1.
\end{align*}
\]

(b) Solve the following non-linear system
\[
\begin{align*}
  2x^2 - y &= 0 \\
  3x - y &= -2.
\end{align*}
\]

(c) Solve the system. Write your answer in a parametric form if necessary.
\[
\begin{align*}
  x + 2y + z &= 4 \\
  x - y + 2z &= 1 \\
  x + 5y &= 7.
\end{align*}
\]

(5 points)