Notes on ICNS 103

Chapter 10: 10.1 Limits

1. The concept of limits involves the notion of getting closer and closer to something, but yet not touching it.

2. We will let a variable "inch up" to a particular value and examine the effect it has on the values of a function.

3. The limit of $f(x)$ as x approaches a is the number L, written

$$\lim_{x \to a} f(x) = L$$

provided that $f(x)$ is arbitrarily close to L for all x sufficiently close to, but not equal to, a. If there is no such number, we say that the limit does not exist.

4. Properties of Limits

(a) If $f(x) = c$ is a constant function, then

$$\lim_{x \to a} f(x) = \lim_{x \to a} c = c$$

(b) $\lim_{x \to a} x^n = a^n$, for any positive integer n

If $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

(c) $\lim_{x \to a} \big(f(x) \pm g(x)\big) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

That is, the limit of a sum or difference is the sum or difference, respectively, of the limits.

(d) $\lim_{x \to a} \big(f(x) \cdot g(x)\big) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

That is, the limit of a product is the product of the limits.

(e) $\lim_{x \to a} \big(cf(x)\big) = c \cdot \lim_{x \to a} f(x)$, where c is a constant

That is, the limit of a constant times a function is the constant times the limit of the function.

Example If f is a polynomial function, then

$$\lim_{x \to a} f(x) = f(a).$$

(f) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

That is, the limit of a quotient is the quotient of limits, provided that the denominator does not have a limit of 0.

(g) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

For this rule, we note that if n is even, we require that $\lim_{x\to a} f(x)$ be positive.

5. If f and g are two functions for which $f(x) = g(x)$, for all $x \neq a$, then

$$\lim_{x\to a} f(x) = \lim_{x\to a} g(x).$$

Examples

(a) Find $\lim_{x\to -1} \dfrac{x^2-1}{x+1}$.

(b) Find $\lim_{x\to 1} \dfrac{x^3-1}{x-1}$.

(c) If $f(x) = x^2 + 1$, find $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$.

6. A special limit:

$$\lim_{x\to 0}(1+x)^{1/x} = e \approx 2.71828.$$

Exercises Do Problems 10.1