1. An antiderivative of a function \( f \) is a function \( F \) such that 
\[ F'(x) = f(x). \]

2. Any two antiderivatives of a function differ only by a constant.

3. The definite integral of any function \( f \) with respect to \( x \) is written 
\[ \int f(x) \, dx \] and denotes the most general antiderivative of \( f \). Hence, 
\[ \int f(x) \, dx = F(x) + C \text{ if and only if } F'(x) = f(x). \]

4. Basic Integration Formulas
   (a) \( \int k \, dx = kx + C; \ k \text{ is a constant} \)
   (b) \( \int x^n \, dx = \frac{1}{n+1}x^{n+1} + C, \ n \neq -1 \)
   (c) \( \int x^{-1} \, dx = \int \frac{1}{x} \, dx = \int \frac{dx}{x} = \ln|x| + C \ \text{ for } x \neq 0 \)
   (d) \( \int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln|ax+b| + C \ \text{ for } a \neq 0 \)
   (e) \( \int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C \ \text{ for } a \neq 0 \)
   (f) \( \int kf(x) \, dx = k \int f(x) \, dx; \ k \text{ is a constant} \)
   (g) \( \int (f(x) \pm g(x)) \, dx = \int f(x) \, dx \pm \int g(x) \, dx \)

5. Techniques of Integration
   (a) Guessing a likely antiderivative
   (b) Substitution
   (c) Integration by parts:
   \[ \int u \, dv = uv - \int v \, du \]
Example Determine the definite integrals.

(a) \( \int \left( \sqrt[3]{x} - \frac{3}{\sqrt[3]{x}} \right) dx \)  
(b) \( \int (e^{2x} - x^3(\sqrt[3]{x} + 1)) dx \)  
(c) \( \int \frac{(x^3 + 1)^2}{x^2} dx \)

(d) \( \int \frac{2x^2}{3 - 4x^3} dx \)  
(e) \( \int \sqrt[3]{x} + 1 dx \)  
(f) \( \int \frac{x^3}{\sqrt{x^2 + 1}} dx \)

(g) \( \int \frac{3x + 5}{e^{2x}} dx \)  
(h) \( \int (x - e^{-x})^2 dx \)  
(i) \( \int x^5e^{x^2} dx \)

Example The sole producer of a product has determined that the marginal-revenue function is \( \frac{dr}{dq} = 100 - 3q^2 \).

Determine the point elasticity of demand for the product when \( q = 5 \).

Example A manufacturer has determined that the marginal-cost function is \( \frac{dc}{dq} = 0.003q^2 - 0.4q + 40 \)

where \( q \) is the number of units produced. If marginal cost is \$27.50\ when \( q = 50 \) and fixed costs are \$5000, what is the average cost of producing 100 units?

Example The marginal-cost function for a manufacturer’s product is given by \( \frac{dc}{dq} = \frac{9}{10} \sqrt[4]{q} \sqrt{0.04q^{3/4} + 4} \)

where \( c \) is the total cost in dollars when \( q \) units are produced. Fixed costs are \$360.\n
(a) Determine the marginal cost when 25 units are produced.  
(b) Find the total cost of producing 25 units.