Notes on ICNS 103

Chapter 10: 10.2 Limits (Continued)

1. Terms to consider: one-sided limits, infinite limits, limits at infinity.

2. The limit exists if and only if both one-sided limits exist and are equal.

Example: One-Sided Limits and Infinite Limits
Find the limit (if it exists).

(a) \[ \lim_{x \to 1^+} \frac{2}{x + 1} \]
(b) \[ \lim_{x \to -2} \frac{x + 2}{x^2 - 4} \]

Example: Limits at Infinity
Find the limit (if it exists).

(a) \[ \lim_{x \to \infty} \frac{4}{(x - 5)^3} \]
(b) \[ \lim_{x \to -\infty} \sqrt{4 - x} \]

3. We note the following:
\[ \lim_{x \to \pm \infty} \frac{1}{x^p} = 0 \quad \text{where } p > 0. \]

4. Limits at Infinity for Rational Functions
If \( f(x) \) is a rational function and \( a_n x^n \) and \( b_m x^m \) are the terms in the numerator and denominator, respectively, with the greatest powers of \( x \), then
\[ \lim_{x \to \pm \infty} f(x) = \lim_{x \to \pm \infty} \frac{a_n x^n}{b_m x^m}. \]

Example
Find the limit (if it exists).

(a) \[ \lim_{x \to \infty} \frac{x^2 - 1}{7 - 2x + 8x^2} \]
(b) \[ \lim_{x \to -\infty} \frac{x}{(3x - 1)^2} \]
(c) \[ \lim_{x \to \infty} \frac{x^5 - x^4}{x^4 - x^3 + 2} \]
5. As \( x \to \infty \) (or \( x \to -\infty \)), the limit of a polynomial function is the same as the limit of its term that involves the greatest power of \( x \). For example, \( \lim_{x \to -\infty} (x^3 - x^2 + x - 2) = \lim_{x \to -\infty} x^3 = -\infty \).

6. **Limits for a Case-Defined Function**

   **Example**

   If \( f(x) = \begin{cases} 
   x^2 + 1 & \text{if } x \geq 1 \\
   3 & \text{if } x < 1 
   \end{cases} \), find the limit (if it exists):

   \[
   \lim_{x \to 1^+} f(x), \quad \lim_{x \to 1^-} f(x), \quad \lim_{x \to 1} f(x), \quad \lim_{x \to \infty} f(x), \quad \lim_{x \to -\infty} f(x).
   \]

**Exercises** Do Problems 10.2