1. A secant line is a line that intersects a curve at two or more points.

2. The tangent line to the curve at $P$ is defined to be the common limiting position of the secant lines joining the point $P$ with any other points of the curve.

3. The slope of a curve at a point $P$ is the slope, if it exists, of the tangent line at $P$.

4. The slope of the tangent line at $(a, f(a))$ is given by

$$m_{\text{tan}} = \lim_{z \to a} \frac{f(z) - f(a)}{z - a} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}.$$

5. The derivative of a function $f$ is the function denoted $f'$ and defined by

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

provided that this limit exists. If $f'(a)$ can be found, then $f$ is said to be differentiable at $a$, and $f'(a)$ is called the derivative of $f$ at $a$ or the derivative of $f$ with respect to $x$ at $a$. The process of finding the derivative is called differentiation.

6. Because the derivative gives the slope of the tangent line, $f'(a)$ is the slope of the line tangent to the graph of $y = f(x)$ at $(a, f(a))$.

7. If $f$ is differentiable at $a$, then $f$ is continuous at $a$. That is, differentiability at a point implies continuity at that point.

8. It is false that continuity implies differentiability. For example, consider the function $f(x) = |x|$. This function is continuous at 0 but not differentiable there.

Examples

(a) Use the definition of the derivative to find each of the following:

1. $f'(x)$ if $f(x) = 4x - 1$
2. $\frac{dp}{dq}$ if $p = 3q^2 + 2q + 1$
3. $\frac{d}{dx}\sqrt{x + 2}$

(b) Find an equation of the tangent line to the curve $y = \frac{3}{x - 1}$ at the point $(2, 3)$.

(c) Find an equation of the tangent line to the curve $y = (x - 7)^2$ at the point $(6, 1)$.

Exercises Do Problems 11.1.