(9)

Notes on ICNS 103

Chapter 12: 12.3 Elasticity of Demand, 12.4 Implicit Differentiation, 12.5 Logarithmic Differentiation, 12.7 Higher-Order Derivatives

1. If $p = f(q)$ is a differentiable demand function, the <u>point elasticity of demand</u>, denoted by the Greek letter η (eta), at (p, q) is <u>given by</u>

$$\eta = \frac{\frac{p}{q}}{\frac{dp}{dq}} = \frac{p}{q}\frac{dq}{dp} \approx \frac{\text{percentage change in quantity}}{\text{percentage change in price}}.$$

2. There are three categories of elasticity:

   (a) When $|\eta| > 1$, demand is <u>elastic</u>.

   (b) When $|\eta| = 1$, demand has <u>unit elasticity</u>.

   (c) When $|\eta| < 1$, demand is <u>inelastic</u>

<u>Example</u> The demand equation for a product is

$$q = 500 - 40p + p^2$$

where p is the price per unit (in dollars) and q is the quantity of units demanded (in thousands). Find the point elasticity of demand when $p = 15$. If this price of 15 is increased by $\frac{1}{2}$%, what is the approximate change in demand?

3. Implicit Differentiation Procedure

   For an equation that we assume defines y implicitly as a differentiable function of x, the derivative $\frac{dy}{dx}$ can be found as follows:

   (a) Differentiate both sides of the equation with respect to x.

   (b) Collect all terms involving $\frac{dy}{dx}$ on one side of the equation, and collect all other terms on the other side.

   (c) Solve for $\frac{dy}{dx}$.

<u>Example</u> Find $\frac{dy}{dx}$ by implicit differentiation.

(a) $3x^2 + 6y^2 = 1$ (b) $x^2 + xy - 2y^2 = 0$

(c) $x = \sqrt{y} + \sqrt[4]{y}$ (d) $(1 + e^{3xy})^2 = 3 + \ln(x - y^2)$

Example Find an equation of the tangent line to the curve

$$x^3 + xy + y^2 = -1$$

at the point $(-1, 1)$.

Example Find the rate of change of q with respect to p for

$$p = \frac{20}{(q+5)^2}.$$

4. Logarithmic Differentiation

To differentiate $y = f(x)$,

(a) Take the natural logarithm of both sides.

(b) Simplify $\ln(f(x))$ by using properties of logarithms and differentiate both sides with respect to x.

(c) Solve for $\frac{dy}{dx}$.

Example Find y' using logarithmic differentiation.

(a) $y = (x+1)^2(x-2)(x^2+3)$ (b) $y = (2x+1)\sqrt[3]{2x+5}\sqrt{x^3+2}$

(c) $y = x^{x^2+1}$ (d) $y = (\ln x)^{e^x}$

Example If $y = (3x)^{-2x}$, find the value of x for which the percentage rate of change of y with respect to x is 60.

Example Find the indicated derivatives.

(a) $y = 4x^3 - 12x^2 + 6x + 2, \quad y'''$ (b) $y = 2x^{1/2} + (2x)^{1/2}, \quad \frac{d^2y}{dx^2}$

(c) $x^2 + 4y^2 - 16 = 0, \quad y''$ (d) $e^x - e^y = x^2 + y^2, \quad \frac{d^2y}{dx^2}$

Example If $c = 0.3q^2 + 2q + 850$ is a cost function, how fast is marginal cost changing when $q = 100$?

Exercises Do Problems 12.3, 12.4, 12.5, 12.7