Solutions to Homework 3

1.

$f(x) = 5$ is a constant function, so $f'(x) = 0$

2.

$$f'(x) = 4(2x) - 2(1) + 0 = 8x - 2$$

3.

$$y = x^{7/2},\ y' = \frac{7}{2}x^{\frac{7}{2}-1} = \frac{7}{2}x^{5/2}$$

4.

$$\Phi(x) = \frac{1}{3}x^3 - 3x^{-3},$$
$$\Phi'(x) = \frac{1}{3}(3x^2) - 3(-3x^{-4}) = x^2 + 9x^{-4}$$

5.

$$y' = 3 - 2x^{-1/2} = 3 - \frac{2}{\sqrt{x}}$$
$$y'\big|_{x=4} = 2$$
$$y'\big|_{x=9} = \frac{7}{3}$$
$$y'\big|_{x=25} = \frac{13}{5}$$

6.

. $y = \dfrac{\sqrt{x}\left(2 - x^2\right)}{x} = x^{-\frac{1}{2}}\left(2 - x^2\right) = 2x^{-\frac{1}{2}} - x^{\frac{3}{2}}.$

$$y' = -x^{-\frac{3}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$
$$y'\big|_{x=4} = -\frac{1}{8} - 3 = -\frac{25}{8}$$

When $x = 4$, then $y = -7$. The tangent line is

$y + 7 = -\dfrac{25}{8}(x - 4)$, or $y = -\dfrac{25}{8}x + \dfrac{11}{2}$.

7.

$$y = x^2 - 5x + 3$$

$$y' = 2x - 5$$

Setting $2x - 5 = 1$ gives $2x = 6$, $x = 3$. When $x = 3$, then $y = -3$. This gives the point $(3, -3)$.

8.

$$g'(x) = (x^{1/2} + 5x - 2)\left(\frac{1}{3}x^{-2/3} - \frac{3}{2}x^{-1/2}\right) + (x^{1/3} - 3x^{1/2})\left(\frac{1}{2}x^{-1/2} + 5\right)$$

$$= \frac{1}{3}x^{-1/6} + \frac{5}{3}x^{1/3} - \frac{2}{3}x^{-2/3} - \frac{3}{2} - \frac{15}{2}x^{1/2} + 3x^{-1/2} + \frac{1}{2}x^{-1/6} + 5x^{1/3} - \frac{3}{2} - 15x^{1/2}$$

$$= \frac{1}{6}(-135x^{1/2} + 40x^{1/3} + 5x^{-1/6} + 18x^{-1/2} - 4x^{-2/3} - 18)$$

9.

$$f'(x) = \frac{(a-x)(1) - (a+x)(-1)}{(a-x)^2} = \frac{2a}{(a-x)^2}$$