

Solutions to Homework 4

1.

$$\begin{aligned}y' &= (x^2 + 3x - 2)(4x - 1) + (2x^2 - x - 3)(2x + 3) \\&= (4x^3 + 12x^2 - 8x - x^2 - 3x + 2) + (4x^3 - 2x^2 - 6x + 6x^2 - 3x - 9) \\&= 8x^3 + 15x^2 - 20x - 7\end{aligned}$$

2.

$$\begin{aligned}\frac{dy}{dx} &= \frac{(4x+1)(2) - (2x-3)(4)}{(4x+1)^2} = \frac{8x+2-8x+12}{(4x+1)^2} \\&= \frac{14}{(4x+1)^2}\end{aligned}$$

3.

$$\begin{aligned}y' &= -\frac{(x-8)(0) - (4)(1)}{(x-8)^2} + \frac{(3x+1)(2) - (2x)(3)}{(3x+1)^2} \\&= \frac{4}{(x-8)^2} + \frac{2}{(3x+1)^2}\end{aligned}$$

4.

$$\begin{aligned}y &= \frac{(9x-1)(3x+2)}{4-5x} = \frac{27x^2+15x-2}{4-5x} \\y' &= \frac{(4-5x)(54x+15) - (27x^2+15x-2)(-5)}{(4-5x)^2} \\&= \frac{-270x^2+141x+60+135x^2+75x-10}{(4-5x)^2} \\&= -\frac{135x-216x-50}{(4-5x)^2}\end{aligned}$$

5.

$$\begin{aligned}y &= \frac{x^3}{x^4+1} \\y' &= \frac{(x^4+1)(3x^2) - (x^3)(4x^3)}{(x^4+1)^2} \\y'(-1) &= \frac{(2)(3) - (-1)(-4)}{(2)^2} = \frac{1}{2}\end{aligned}$$

6.

$$\bar{c} = 0.01q + 5 + \frac{500}{q}$$

$$c = \bar{c}q = 0.01q^2 + 5q + 500$$

$$\frac{dc}{dq} = 0.02q + 5$$

$$\left. \frac{dc}{dq} \right|_{q=50} = 6$$

$$\left. \frac{dc}{dq} \right|_{q=100} = 7$$

7.

$$c = f(q) = 0.4q^2 + 4q + 5$$

$$\frac{dc}{dq} = 0.8q + 4$$

If $q = 2$, then $\frac{dc}{dq} = 5.6$. Over the interval $[2, 3]$,

$$\frac{\Delta c}{\Delta q} = \frac{f(3) - f(2)}{3 - 2} = \frac{20.6 - 14.6}{1} = 6.$$

8.

$$c = 0.3q^2 + 3.5q + 9$$

$$\frac{dc}{dq} = 0.6q + 3.5$$

If $q = 10$, then $\frac{dc}{dq} = 0.6(10) + 3.5 = 9.5$. If

$q = 10$, then $c = 74$ and

$$\frac{\frac{dc}{dq}}{c}(100) = \frac{9.5}{74}(100) \approx 12.8\%.$$

9.

a. $\frac{dr}{dq} = 30 - 0.6q$

b. If $q = 10$, $\frac{r'}{r} = \frac{30 - 6}{300 - 30} = \frac{24}{270} = \frac{4}{45} \approx 0.09$.

c. 9%