Solutions to Homework 5

1.

$\frac{dz}{ds} = \frac{dz}{du} \cdot \frac{du}{ds} = \left(2u + \frac{1}{2\sqrt{u}}\right)(4s)$. If $s = -1$, then

$u = 1$, so $\left.\frac{dz}{ds}\right|_{s=-1} = \left(\frac{5}{2}\right)(-4) = -10$

2.

$$y = \frac{1}{\left(x^2 - 3x\right)^2} = \left(x^2 - 3x\right)^{-2}$$

$$y' = -2\left(x^2 - 3x\right)^{-3}(2x-3)$$

$$= -2(2x-3)\left(x^2 - 3x\right)^{-3}$$

3.

$$y' = 3\left(x^2 - 7x - 8\right)^2(2x-7)$$

If $x = 8$, then slope $= y' = 3(64-56-8)^2(16-7) = 0$.

4.

$$\frac{dc}{dq} = \frac{\left(q^2+3\right)^{\frac{1}{2}}(10q) - \left(5q^2\right)\left[\frac{1}{2}\left(q^2+3\right)^{-\frac{1}{2}}(2q)\right]}{q^2+3}$$

Multiplying numerator and denominator by $\left(q^2+3\right)^{\frac{1}{2}}$ gives

$$\frac{dc}{dq} = \frac{\left(q^2+3\right)(10q) - 5q^2(q)}{\left(q^2+3\right)^{\frac{3}{2}}} = \frac{5q^3+30q}{\left(q^2+3\right)^{\frac{3}{2}}} = \frac{5q\left(q^2+6\right)}{\left(q^2+3\right)^{\frac{3}{2}}}.$$

5.

$$y = x^2 \log_2 x = x^2 \cdot \frac{\ln x}{\ln 2} = \frac{1}{\ln 2}(x^2 \ln x)$$

$$\frac{dy}{dx} = \frac{1}{\ln 2}\left[x^2\left(\frac{1}{x}\right) + \ln x(2x)\right]$$

$$= \frac{x}{\ln 2}(1 + 2\ln x)$$

6.

$$y = \ln\left[\left(x^2+2\right)^2\left(x^3+x-1\right)\right]$$
$$= 2\ln\left(x^2+2\right)+\ln\left(x^3+x-1\right)$$
$$\frac{dy}{dx} = 2\cdot\frac{1}{x^2+2}(2x)+\frac{1}{x^3+x-1}\left(3x^2+1\right)$$
$$= \frac{4x}{x^2+2}+\frac{3x^2+1}{x^3+x-1}$$

7.

$$f'(r) = e^{3r^2+4r+4}(6r+4) = 2(3r+2)e^{3r^2+4r+4}$$

8.

$$y' = 3x^4\left[e^{-x}(-1)\right]+e^{-x}(12x^3) = 3x^3e^{-x}(4-x)$$

9.

$$f(x) = 10^{-x}+\ln(8+x)+0.01e^{x-2}$$
$$= e^{(\ln 10)(-x)}+\ln(8+x)+0.01e^{x-2}$$
$$f'(x) = e^{(\ln 10)(-x)}(-\ln 10)+\frac{1}{8+x}+0.01e^{x-2}$$
$$= -(\ln 10)10^{-x}+\frac{1}{8+x}+0.01e^{x-2}$$
$$\frac{f'(2)}{f(2)} = \frac{-(\ln 10)10^{-2}+\frac{1}{10}+0.01}{10^{-2}+\ln(10)+0.01}\approx 0.0374$$