Solutions to Homework 7

1.

Decreasing on $(-\infty, -1)$ and $(3, \infty)$; increasing on $(-1, 3)$; relative minimum $(-1, -1)$; relative maximum $(3, 4)$.

2.

$y = 2x^3 + 1$

$y' = 6x^2$

CV: $x = 0$

$+ \quad +$

0

Increasing on $(-\infty, 0)$; increasing on $(0, \infty)$; no relative maximum or minimum

3.

$y = 4x^2 + \frac{1}{x}$

$y' = 8x - \frac{1}{x^2} = \frac{(2x-1)(4x^2+2x+1)}{x^2}$

CV: $x = \frac{1}{2}$, but $x = 0$ must be included in the sign chart because it is a point of discontinuity of y.

$- \quad - \quad +$

$\boxed{0} \quad \frac{1}{2}$

Increasing on $\left(\frac{1}{2}, \infty\right)$; decreasing on $(-\infty, 0)$ and $\left(0, \frac{1}{2}\right)$; relative minimum when $x = \frac{1}{2}$.

4.

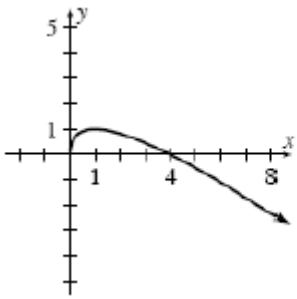
$y = 2\sqrt{x} - x = \sqrt{x}\left(2 - \sqrt{x}\right)$. [Note: $x \geq 0$.]

Intercepts $(0, 0)$, $(4, 0)$

$y' = \frac{1}{\sqrt{x}} - 1 = \frac{1-\sqrt{x}}{\sqrt{x}}$

CV: $x = 0, 1$

Increasing on $(0, 1)$; decreasing on $(1, \infty)$; relative maximum when $x = 1$.



5.

$f(x) = x^2 - 2x + 3$ and f is continuous over $[0, 3]$.

$f'(x) = 2x - 2 = 2(x-1)$

The only critical value on $(0, 3)$ is $x = 1$. We evaluate f at this point and at the endpoints: $f(0) = 3$, $f(1) = 2$, and $f(3) = 6$.

Absolute maximum: $f(3) = 6$;

absolute minimum: $f(1) = 2$

6.

$f(x) = \frac{1}{4}x^4 - \frac{3}{2}x^2$ and f is continuous over $[0, 1]$.

$f'(x) = x^3 - 3x = x\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$

There are no critical values on $(0, 1)$, so we only have to evaluate f at the end points: $f(0) = 0$ and $f(1) = -\frac{5}{4}$

Absolute maximum: $f(0) = 0$;

absolute minimum: $f(1) = -\frac{5}{4}$

7.

$f(x) = \frac{x}{x^2+1}$ and f is continuous over $[0, 2]$.

$f'(x) = \frac{\left(x^2+1\right) - x(2x)}{\left(x^2+1\right)^2} = \frac{1-x^2}{\left(x^2+1\right)^2}$

$= \frac{(1+x)(1-x)}{\left(x^2+1\right)^2}$

The only critical value on $(0, 2)$ is $x = 1$. We have $f(0) = 0$, $f(1) = \frac{1}{2}$, and $f(2) = \frac{2}{5}$.

Absolute maximum: $f(1) = \frac{1}{2}$;

absolute minimum: $f(0) = 0$

8.

$y = -2x^2 + 4x$

$y' = -4x + 4$

$y'' = -4 < 0$ for all x, so the graph is concave down for all x, that is, on $(-\infty, \infty)$.

Solutions to Homework 7

9.

$$y = \frac{7}{x^3} = 7x^{-3}$$

$$y' = -21x^{-4}$$

$$y'' = 84x^{-5} = \frac{84}{x^5}$$

Although y'' is not defined when $x = 0$, y is not continuous there. Thus there is no possible inflection point. However, $x = 0$ must be considered in concavity analysis. Concave down on $(-\infty, 0)$; concave up on $(0, \infty)$; no inflection point

10.

$$y = 3xe^x$$

$$y' = 3xe^x + 3e^x = 3e^x(x+1)$$

$$y'' = 3e^x(1) + 3(x+1)e^x = 3e^x(x+2)$$

$y'' = 0$ if $x = -2$. Concave down on $(-\infty, -2)$; concave up on $(-2, \infty)$; inflection point when $x = -2$.