Problem 13.1

1. Decreasing on \((-\infty, -1)\) and \((3, \infty)\); increasing on \((-1, 3)\); relative minimum \((-1, -1)\); relative maximum \((3, 4)\).

9. \(y = 2x^3 + 1\)
   
   \(y' = 6x^2\)
   
   CV: \(x = 0\)
   
   + +
   
   0

   Increasing on \((-\infty, 0)\); increasing on \((0, \infty)\); no relative maximum or minimum

36. \(y = 4x^2 + \frac{1}{x}\)
   
   \(y' = 8x - \frac{1}{x^2} = \frac{(2x-1)(4x^2 + 2x+1)}{x^2}\)
   
   CV: \(x = \frac{1}{2}\), but \(x = 0\) must be included in the sign chart because it is a point of discontinuity of \(y\).
   
   + +
   
   0 \frac{1}{2}

   Increasing on \(\left(\frac{1}{2}, \infty\right)\); decreasing on \((-\infty, 0)\)

   and \(\left(0, \frac{1}{2}\right)\); relative minimum when \(x = \frac{1}{2}\).
Problem 13.2

1. \( f(x) = x^2 - 2x + 3 \) and \( f \) is continuous over \([0, 3]\).
   \[ f'(x) = 2x - 2 = 2(x - 1) \]
   The only critical value on \((0, 3)\) is \( x = 1 \). We evaluate \( f \) at this point and at the endpoints:
   \( f(0) = 3, f(1) = 2, \) and \( f(3) = 6 \).
   Absolute maximum: \( f(3) = 6 \);
   absolute minimum: \( f(1) = 2 \)

4. \( f(x) = \frac{1}{4} x^4 - \frac{3}{2} x^2 \) and \( f \) is continuous over \([0, 1]\).
   \[ f'(x) = x^3 - 3x = x(x + \sqrt{3})(x - \sqrt{3}) \]
   There are no critical values on \((0, 1)\), so we only have to evaluate \( f \) at the endpoints:
   \( f(0) = 0 \) and
   \[ f(1) = -\frac{5}{4} \]
   Absolute maximum: \( f(0) = 0 \);
   absolute minimum: \( f(1) = -\frac{5}{4} \)
12. \( f(x) = \frac{x}{x^2 + 1} \) and \( f \) is continuous over \([0, 2]\).

\[
f'(x) = \frac{(x^2 + 1) - x(2x)}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}
\]

\[
= \frac{(1 + x)(1 - x)}{(x^2 + 1)^2}
\]

The only critical value on \((0, 2)\) is \(x = 1\). We have \(f(0) = 0\), \(f(1) = \frac{1}{2}\), and \(f(2) = \frac{2}{5}\).

Absolute maximum: \(f(1) = \frac{1}{2}\);
Absolute minimum: \(f(0) = 0\)

Problem 13.3

7. \( y = -2x^2 + 4x \)
   \( y' = -4x + 4 \)
   \( y'' = -4 < 0 \) for all \(x\), so the graph is concave down for all \(x\), that is, on \((-\infty, \infty)\).

16. \( y = \frac{7}{x^3} = 7x^{-3} \)
   \( y' = -21x^{-4} \)
   \( y'' = 84x^{-5} = \frac{84}{x^5} \)

Although \( y'' \) is not defined when \(x = 0\), \(y\) is not continuous there. Thus there is no possible inflection point. However, \(x = 0\) must be considered in concavity analysis. Concave down on \((-\infty, 0)\); concave up on \((0, \infty)\); no inflection point

31. \( y = 3xe^x \)
   \( y' = 3xe^x + 3e^x = 3e^x(x+1) \)
   \( y'' = 3e^x(l) + 3(x+1)e^x = 3e^x(x+2) \)
   \( y'' = 0 \) if \(x = -2\). Concave down on \((-\infty, -2)\); concave up on \((-2, \infty)\); inflection point when \(x = -2\).