

# What Is the Value of Limited Feedback for MIMO Channels?

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## ABSTRACT

Feedback in a communications system can enable the transmitter to exploit channel conditions and avoid interference. In the case of a multiple-input multiple-output channel, feedback can be used to specify a precoding matrix at the transmitter, which activates the strongest channel modes. In situations where the feedback is severely limited, important issues are how to quantize the information needed at the transmitter and how much improvement in associated performance can be obtained as a function of the amount of feedback available. We give an overview of some recent work in this area. Methods are presented for constructing a set of possible precoding matrices, from which a particular choice can be relayed to the transmitter. Performance results show that even a few bits of feedback can provide performance close to that with full channel knowledge at the transmitter.

## INTRODUCTION

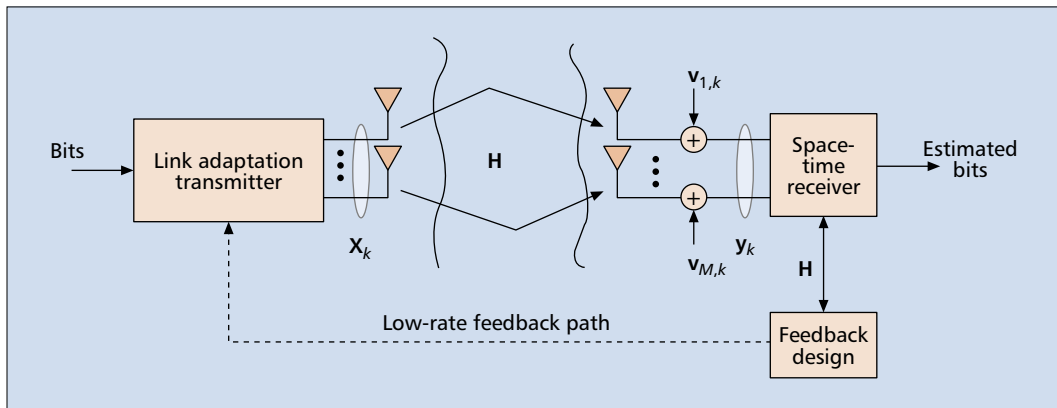
Multiple antennas, when used at both the transmitter and the receiver, create a multiple-input multiple-output (MIMO) propagation channel. Using sophisticated coding at the transmitter and substantial signal processing at the receiver, the MIMO channel can be provisioned for higher data rates, resistance to multipath fading, lower delays, and support for multiple users. Current research efforts demonstrate that MIMO technology has great potential in third- and fourth-generation (3G, 4G) cellular systems, fixed wireless access, wireless local area networks, and ad hoc wireless battlefield networks.

Optimizing MIMO networks using channel state information at the transmitter (often called closed-loop MIMO communication) can help customize the transmitted waveforms to provide higher link capacity and throughput, improve system capacity by sharing the spatial channel with multiple users simultaneously, enable channel-aware scheduling for multiple users, simplify multi-user receivers through interference avoid-

ance, and provide a simple and general means to exploit spatial diversity. Essentially, channel state information makes it easier to obtain the benefits of MIMO technology while lessening the complexity impact incurred through MIMO transmission and reception.

A consequence of using multiple antennas, however, is an increase in the number of channel state parameters. Training can be used to estimate the channel at the receiver. In some cases the transmit channel can be inferred from the receive channel, but more often channel state information needs to be quantized and sent to the transmitter over a limited-rate feedback channel. This is not unreasonable; control channels are often available to implement power control, adaptive modulation, and certain closed-loop diversity modes (e.g., in 3G). Unfortunately, the feedback requirements in a MIMO system generally grow with the product of the transmit antennas, receive antennas, delay spread, and number of users, while the capacity only grows linearly. For example, a complex four-transmit and four-receive matrix channel has 32 parameters that must be quantized every time the channel changes. Compared with the 1 parameter needed for fast power control in a single antenna link, this is an increase over a factor of 30! Clearly, there needs to be a better approach.

In this article we review one promising solution to this problem known as *limited feedback communication*. The basic idea is to use intelligent vector quantization (VQ) techniques to quantize channel state information prior to transmission over a limited data rate feedback channel. This entails designing a codebook that encapsulates the essential degrees of freedom of the channel and is tailored to the channel model and receiver design. The fundamental difference with traditional VQ lies in the choice of distortion measures. A pure VQ approach would attempt to obtain a good approximation of a given channel realization; the goal of limited feedback communication, though, is to maximize capacity or minimize bit error rate with a few bits of feedback information. Thus, it is not the



■ **Figure 1.** A block diagram of a limited feedback MIMO system.

reconstruction of the channel that is of interest, but achieving a good approximation of what might be done with that channel. The application of such quantization techniques to MIMO communication is a rich area for algorithm development and associated performance analysis. In this article we attempt to review the state of the art in limited feedback communication for MIMO communication systems. We review prior work in the area, as well as related work on transmit diversity. We consider the general VQ model and provide examples of the performance benefits of low-rate designs that have potential application in a variety of MIMO communication scenarios. Finally, we point out future directions for research based on what is currently practical.

## LINEAR PRECODING

Narrowband MIMO systems with  $M_t$  transmit and  $M_r$  receive antennas experience a channel that can be modeled as an  $M_r \times M_t$  matrix  $\mathbf{H}$ . In wireless systems the channel is well modeled as a *random matrix*. Common random matrix models for this channel include uncorrelated Rayleigh fading (i.e., the entries of  $\mathbf{H}$  are independent and identically distributed, *i.i.d.*, complex normal random variables), correlated Rayleigh fading, uncorrelated Rician fading, and correlated Rician fading [1].

Most work on closed-loop MIMO channels has concentrated on linearly precoded space-time block codes [1]. Single-user linearly precoded space-time block codes are described by the input/output relationship

$$\mathbf{Y} = \mathbf{H}\mathbf{F}\mathbf{S} + \mathbf{V}, \quad (1)$$

where  $\mathbf{F}$  is an  $M_t \times M$  precoding matrix,  $\mathbf{S}$  is an  $M \times T$  space-time block codeword, and  $\mathbf{V}$  is an  $M_r \times T$  noise matrix [1]. The precoder parameter  $M$  is chosen so that  $M \leq M_t$ . The space-time block codeword (whether it be spatial multiplexing, orthogonal space-time block coding, etc.) is generated *independent of the channel*. Although not discussed in this article, note that varying the transmission rate as a function of channel conditions can add performance improvements [2]. The only form of link adaptation considered in this article arises from the precoding matrix  $\mathbf{F}$ . The precoder is chosen using a function  $f$  that maps an  $M_r \times M_t$  channel realization to an  $M_t \times M$  precoding matrix with  $\mathbf{F} = f(\mathbf{H})$ .

The general input/output relationship in Eq. 1 covers a large range of closed-loop MIMO techniques. These include the popular beamformers that convert a MIMO channel into an equivalent single-input single-output (SISO) channel, precoded spatial multiplexing, and precoded orthogonal space-time block codes [1]. It also includes the antenna selection techniques where  $M$  out of  $M_t$  antennas are selected for transmission. In that case, the matrix  $\mathbf{F}$  consists of  $M$  different columns of the  $M \times M_t$  identity matrix.

The matrix  $\mathbf{F}$  *adapts* the transmitted signal to the current channel conditions. For this reason, the transmitter must have some knowledge of the channel when designing  $\mathbf{F}$ . There has been much work recently on the design and performance of precoding methods under different assumptions about what information is available at the transmitter. For example, these assumptions include perfect channel knowledge [1, 3, references therein], incomplete or estimated knowledge of subspaces associated with the channel [4, 5], and statistical channel knowledge (e.g., [6–8]). Given a low-rate feedback channel with frequency-division duplexing, full, or accurate but incomplete, channel knowledge may be difficult to obtain at the transmitter. Statistical feedback of spatial correlations may be helpful when the channel varies rapidly, but cannot be used to exploit strong channel modes associated with a static or slowly varying channel. Therefore, it is of great interest to find efficient ways of designing  $\mathbf{F}$  based on current channel conditions.

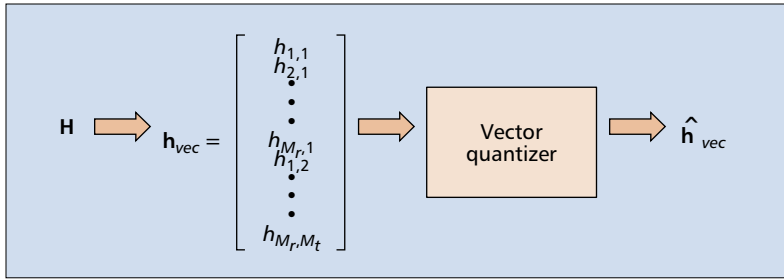
## LIMITED FEEDBACK COMMUNICATION

The design of limited feedback MIMO systems represents a nontrivial problem, with the potential for substantial performance gains. In this section we give an overview of the general area of limited feedback MIMO systems.

### SYSTEM OVERVIEW

Employing limited feedback in coherent MIMO communication systems requires cooperation between the transmitter and receiver. A general overview of this cooperation in a narrowband system is illustrated in Fig. 1. The receiver uses its estimate of the forwardlink channel matrix  $\mathbf{H}$  to design feedback that the transmitter can use to adapt the transmitted signal to the channel.

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■ **Figure 2.** An illustration of channel quantization.

Note that while this model is specific to a flat-fading channel, it easily extends to a frequency-selective channel model if the system uses orthogonal frequency-division multiplexing (OFDM). A MIMO system using OFDM (often denoted MIMO-OFDM) divides a large band into small narrowband channels using an orthogonal transformation. Assuming that the MIMO-OFDM system has been designed correctly, signals sent on each narrowband channel will experience flat fading. Thus, the limited feedback techniques designed for narrowband systems can be successfully used in MIMO-OFDM systems.

There are two main approaches to designing feedback: quantizing the channel or quantizing properties of the transmitted signal. We will discuss the ideas behind both of these techniques. For most closed-loop signaling schemes, either method can be employed. It will be apparent, however, that channel quantization offers an intuitively simple approach to closed-loop MIMO, but lacks the performance of more specialized feedback methods.

### CHANNEL QUANTIZATION

The fundamental idea behind closed-loop MIMO is to adapt the transmitted signal to the channel. One approach to limited feedback, suggested by the large body of VQ work, is to employ channel quantization, which is illustrated in Fig. 2. This problem is reformulated as a VQ problem by stacking the columns of the channel matrix  $\mathbf{H}$  into an  $M_r \cdot M_t$  dimensional complex vector  $\mathbf{h}_{vec}$ . The vector  $\mathbf{h}_{vec}$  is then quantized using a VQ algorithm.

A vector quantizer works by mapping a real or complex valued vector into one of a finite number of vector realizations. The mapping is usually designed to minimize some sort of distortion function such as the average mean squared error (MSE) between the input vector and the quantized vector. The key difference between channel VQ and VQ discussed in the compression literature is that in the former case, the cost function can exploit any channel invariance, which may be present in the communication system. For example, Narula *et al.* noticed in [9] that closed-loop beamforming is invariant to the channel being multiplied by  $e^{j\theta}$  for any  $\theta$ . This invariance was used to derive a phase-invariant MSE distortion function that reduces the number of feedback parameters required.

Sending a quantized version of the forward link channel from receiver to transmitter gives the transmitter more flexibility to choose among different space-time signaling techniques. In par-

ticular, channel quantization has been employed for multiple-input single-output (MISO) beamforming [9, 10] and MIMO precoded orthogonal space-time block codes [11].

### QUANTIZED SIGNAL ADAPTATION

The work in [9] motivated a new approach to limited feedback MIMO communications. While the algorithm in [9] was still, in some sense, quantizing a MISO vector channel (i.e., multiple transmit antennas and one receive antenna), it was also quantizing the optimal beamforming vector. This subtle difference raises an important question. *Why should the entire channel be quantized when only a portion of the channel structure is needed?*

The answer is that for a fixed transmission technique, performance gains can be achieved by focusing on improving the quantized information needed to adapt the transmitted signal to current channel conditions. In particular, research has concentrated on enhancing the precoded space-time block coding model described by Eq. 1 to account for quantized signal adaptation. These methods are often only a function of the channel singular vectors, thus yielding a dramatic reduction in the dimensionality of the quantization problem.

Limited feedback precoding restricts the selection function  $f$  (where  $\mathbf{F} = f(\mathbf{H})$ ), so  $f$  maps to a codebook

$$\mathcal{F} = \{\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_N\} \quad (2)$$

of possible precoding matrices. The value of  $N$  in Eq. 2 is defined such that  $N = 2^B$  for an integer  $B$ . The chosen matrix can then be conveyed from the receiver to transmitter using  $B$  bits of feedback. This model has been proposed for limited feedback beamforming [12, 13], precoded orthogonal space-time block codes [1, 14], precoded spatial multiplexing [15, 16], and transmit covariance optimization [17, 18].

System performance is closely coupled to the precoder selection function  $f$  and precoder codebook  $\mathcal{F}$ . Selection functions have been proposed to minimize some bound on the probability of error [1, 12–15]. The design of the codebook, however, is a much more difficult problem. The reason is that the distribution of the channel matrix  $\mathbf{H}$  and the selection function must be taken into account. Results in [12–15] have found that in uncorrelated Rayleigh fading the problem relates to designing matrix codes with maximally spaced subspaces. In particular, the codebooks are designed so that  $\min_{1 \leq k < l < N} d(\mathbf{F}_k, \mathbf{F}_l)$  is maximized where  $d(\mathbf{F}_k, \mathbf{F}_l)$  is a *subspace distance*. Subspace distances are only a function of the subspaces spanned by the columns of  $\mathbf{F}_k$  and  $\mathbf{F}_l$ , respectively. Subspace distances can be defined in a number of different ways and are dependent on the dimension  $M$  chosen for the precoder matrix.

Intuitively, one might expect that a *random* selection of matrices in the codebook  $\mathcal{F}$  is likely to result in a large subspace distance between any pair of matrices in the codebook. This intuition is valid for a large number of antennas  $M_t$ , and is related to the fact that two vectors with i.i.d. components become orthogonal (with probability one) as the length becomes large. In the case of a random MIMO channel with i.i.d. components, the columns of the optimal precoding matrix are

eigenvectors of the channel covariance matrix, which are isotropically distributed. These considerations motivated the Random VQ (RVQ) scheme proposed in [16], in which the elements of the codebook  $\mathbf{F}$  are independently chosen random unitary matrices (i.e.,<sup>1</sup>  $\mathbf{F}_k^H \mathbf{F}_k = \mathbf{I}$  for each  $k$ ).

When used for beamforming in a MISO channel, RVQ is asymptotically optimal in the sense that it achieves the maximum rate over any codebook. Furthermore, the asymptotic achievable rate can be explicitly computed for both MISO and MIMO channels [16]. Here asymptotic means for a large system in which the number of antennas  $M_t$  and  $M_r$  each go to infinity with fixed ratio (or in the MISO case  $M_r$  goes to infinity), while also fixing  $B/M_t M_r$ , the number of feedback bits per dimension. A random beamforming scheme is also analyzed for the cellular downlink in [19].

### SCALAR QUANTIZATION SCHEMES

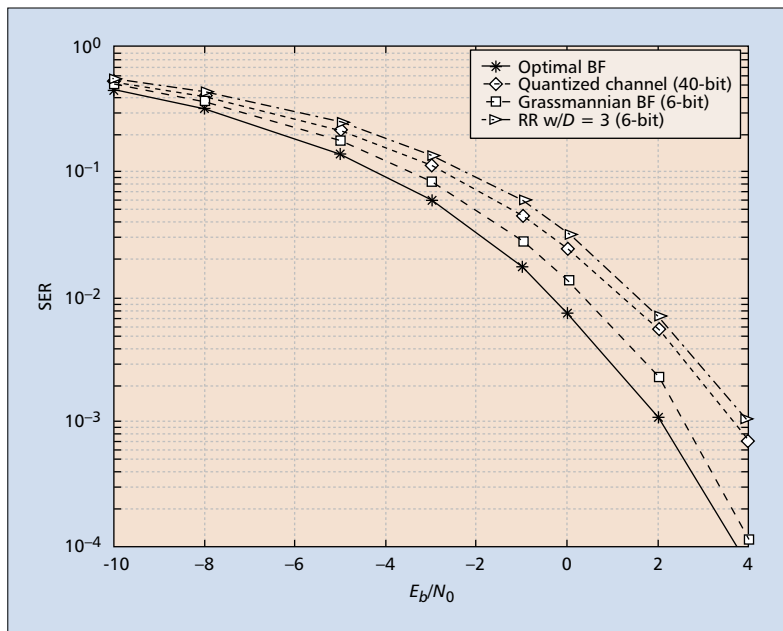
A drawback of VQ schemes is complexity. Namely, in general the receiver must select a precoding matrix from among the  $2^B$  possibilities via an exhaustive search. This clearly becomes a large computational burden as  $B$  increases. When  $B$  is sufficiently large, the precoding matrix can be accurately specified through scalar quantization of the matrix elements. For moderate values of  $B$  VQ may be too complicated, however, and scalar quantization may perform poorly (e.g., when  $B < 2MM_t$ , so there are less than 2 b/complex precoding matrix element).

One approach to improving the performance of scalar quantization when  $B$  is small is to constrain the columns of the precoding matrix to lie in a lower-dimensional subspace with dimension  $D < M_t$ . In this way the feedback bits are distributed over a smaller number of coefficients, which can be represented more accurately. This *reduced rank* approach was proposed in [20] for signature optimization in a code-division multiple access (CDMA) system. In this case the signature (vector) for a particular user is constrained to lie in a lower-dimensional subspace.

To illustrate, consider the beamforming scenario where each  $\mathbf{F}_n$  in the codebook is a rank-one matrix specified by the  $M_t \times 1$  vector  $\mathbf{f}_n$  (i.e.,  $\mathbf{F}_n = \mathbf{f}_n \mathbf{f}_n^H$ ). If  $\mathbf{f}_n$  lies in a  $D$ -dimensional subspace, where  $D \leq M_t$ , we can write  $\mathbf{f}_n = \mathbf{P}_n \alpha_n$  where  $\mathbf{P}_n$  is an  $M_t \times D$  orthogonal matrix, the columns of which span the  $D$ -dimensional subspace, and  $\alpha_n$  is the  $D \times 1$  vector of combining coefficients. The matrix  $\mathbf{P}_n$  is known to the transmitter a priori, so the receiver must compute the optimal set of  $DM$  coefficients (by solving an eigenvector problem), quantize them (using a simple scalar quantizer for each coefficient), and relay them back to the transmitter. Varying the subspace dimension  $D$  allows a trade-off between the available degrees of freedom for precoding and quantization accuracy. Namely, for small  $D$  the performance is limited by the subspace constraint, whereas for large  $D$  the performance is limited by quantization accuracy. In general, the dimension  $D$  can be optimized for a given number of feedback bits  $B$ .

### PERFORMANCE RESULTS

The benefit of limited feedback is illustrated in three different performance plots, generated by Monte Carlo simulations.



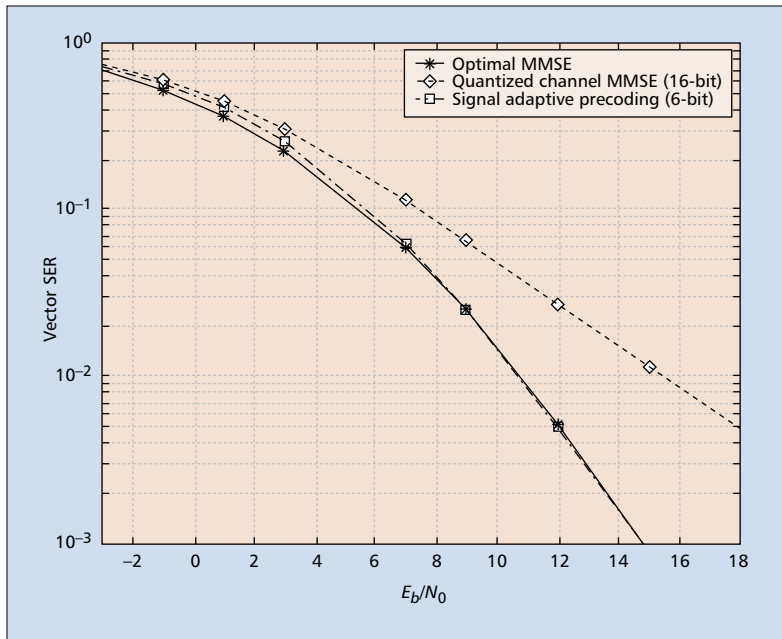
■ **Figure 3.** Limited feedback beamformer performance for a four-transmit five-receive antenna system.

The first plot in Fig. 3 shows the symbol error rate performance of a four-transmit five-receive antenna beamforming system transmitting 16-quadrature amplitude modulation (QAM). Optimal maximum ratio combining is used at the receiver. Signal adaptive beamforming using a 6-bit VQ codebook designed with the criterion in [12] outperforms 40-bit (2 b/complex entry) channel quantization by approximately 1 dB. Limited feedback signal adaptive beamforming also performs within 0.7 dB of full-transmit-channel-knowledge unquantized beamforming. Also shown in Fig. 3 is the performance of a reduced-rank beamformer with dimension  $D = 3$ , quantized with 6 bits, or 2 b/complex coefficient. The performance is comparable to 40-bit channel quantization, and is about 1 dB worse than signal adaptive VQ.

The reason for the dramatic performance gains with the limited feedback signal adaptive approach over channel quantization is because the quantization problem focuses *strictly on the singular vector structure of the channel*. The 40-bit channel quantization has such large quantization error that the fragile eigenstructure of the channel is often mangled at the transmitter. The lack of reliable eigenstructure information at the transmitter causes a loss in performance for the beamformer.

Figure 4 compares the vector symbol error rate (the probability that at least one symbol is in error) of two substream (i.e.,  $M = 2$ ) spatial multiplexing precoders in a four-transmit two-receive antenna system. Signal adaptive limited feedback precoding with a 6-bit codebook designed using techniques from [15] is compared with precoding using 16-bit channel quantization (2 b/complex entry). Unquantized minimum MSE precoding using a maximum singular value power constraint was simulated. Note that limited feedback signal adaptive precoding provides more than a 4 dB gain over channel quantization

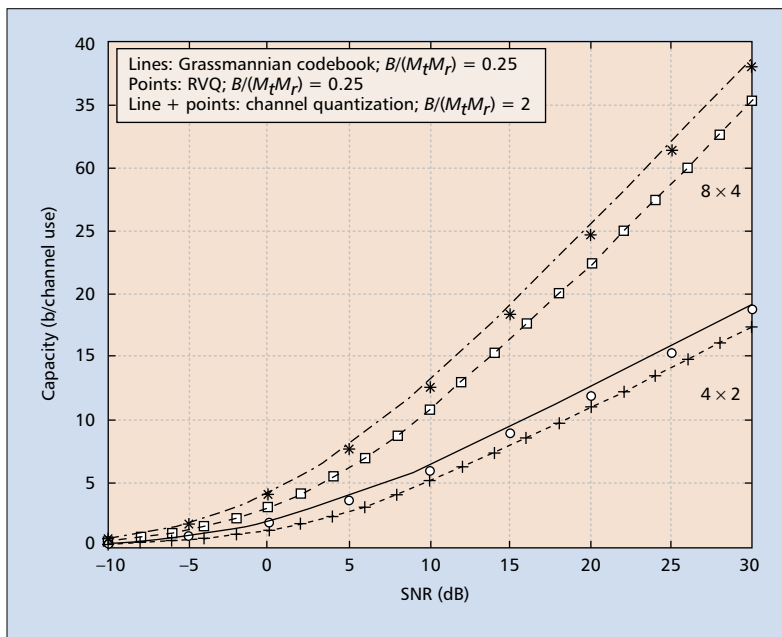
<sup>1</sup> A superscript  $\dagger$  is used to denote the conjugation and transposition of a matrix.



■ **Figure 4.** Limited feedback precoding performance for a four-transmit two-receive antenna system.

at an error rate of  $10^{-2}$  and performs nearly as well as an optimal beamformer with perfect channel knowledge.

Once again, this performance difference arises from the inability of direct channel quantization to capture the eigenstructure of the channel. Direct channel quantization provides the transmitter with an unreliable estimate of the singular values and singular vectors. Limited feedback signal adaptation does not suffer from this problem because it focuses on quantizing the information necessary to design a high-performance precoder.



■ **Figure 5.** Ergodic channel capacity with limited feedback. Results are shown for two systems: four transmit and two receive antennas; and eight transmit and four receive antennas.

The final plot, shown in Fig. 5, illustrates the performance of limited feedback precoding when combined with channel coding. Namely, the performance measure is channel capacity, which for the MIMO channel in Fig. 1, described by the input/output relationship of Eq. 1, is the maximum mutual information between the transmitted symbols  $\mathbf{S}$  and output  $\mathbf{Y}$ , and is given by

$$I(\mathbf{F}) = \log \det \left( \mathbf{I} + \frac{\rho}{M} \mathbf{H} \mathbf{F} \mathbf{F}^\dagger \mathbf{H}^\dagger \right) \quad (3)$$

in bits per channel use, where  $\rho$  is the signal-to-noise ratio (SNR). (The transmitted symbols across antennas are assumed to be uncorrelated.) The receiver therefore selects the matrix  $\mathbf{F}_k$  in the VQ codebook to maximize  $I(\mathbf{F}_k)$ .

Figure 5 shows ergodic channel capacity (the mutual information averaged over channel realizations) for the cases where the precoding matrix is selected via VQ and channel quantization with  $2M_t M_r$  feedback bits (2 b/complex entry). Here results are shown corresponding to VQ using the criterion in [15] and RVQ, both with  $0.25 M_t M_r$  feedback bits. The VQ schemes achieve similar rates and require much less feedback than channel quantization, which achieves a lower rate.

### COMMERCIAL APPLICATION

Limited feedback techniques have already been considered in 3G cellular standards. These techniques are available for use by the transmit adaptive array (TXAA) mode in the closed-loop portion of the 3G Partnership Project (3GPP) standard [10], specifically closed-loop diversity mode design for two transmit antennas.

These 3GPP methods actually represent both channel quantization and quantized signal adaptation approaches. Feedback design in 3GPP systems is based on two cases, quantized phase information (mode 1) and direct channel quantization (mode 2). The quantized phase algorithm actually uses a set number of bits to quantize the phase angles needed to perform equal gain beamforming (i.e., forcing the entries of the beamforming vector to have equal magnitude) at the transmitter. The direct channel quantization allocates a set number of bits for the gain and phase portions of each channel entry, as opposed to the more sophisticated VQ techniques described above.

The gains of mode 1 over the open loop diversity mode, which is a variation of the Alamouti transmit diversity scheme, are around 1 dB with a good feedback channel. Mode 2 has a gain closer to 2 dB. Closed-loop techniques typically work better in slower changing propagation environments since in these cases it is easier to keep up with variations in the channel.

### CONCLUSIONS AND FUTURE DIRECTIONS

This article outlines a general framework for enabling limited feedback in closed-loop MIMO systems. We review the application of limited feedback to MIMO communication and discuss the design of appropriate codebooks. Numerical examples illustrate that relatively little feedback can provide substantial performance improvements.

The impact of limited feedback on MIMO systems will not be felt commercially until the practical effects of limited feedback are fully understood. Channel estimation error and channel evolution will definitely compromise expected performance improvements, but simulations and experimental results are required to determine how “recent” the feedback bits must be to maintain satisfactory performance. More work is also needed in the area of limited feedback applications in MIMO-OFDM systems. While narrowband analysis can easily be applied, the amount of feedback,  $B$  bits for each of  $N$  tones, could be overwhelming. A more practical technique is to feed back information on a select subset of tones and then use interpolation techniques. Other applications of limited feedback such as for multi-user MIMO channels are promising areas for investigation.

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