Minimum-Energy Bit Allocation Over Powerline OFDM Channels

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Abstract—We propose two bit allocations for a powerline OFDM channel to minimize a total transmission energy. The first is the optimal bit allocation derived for a deterministic channel model. The optimal number of bits allotted for each subchannel depends on channel gains, required data rate and transmission duration. For the second allocation, an equal number of bits is transmitted on a subchannel whose ratio between a noise power and squared channel gain is less than certain threshold. This on-off threshold-based bit allocation is also analyzed for a random channel model for which the closed-form expression for the total energy consumption is obtained in a large system limit. Numerical examples show that the optimal bit allocation could results in 83% reduction in transmission energy, compared to a uniform bit allocation and that the on-off allocation performs close to the optimum when a transmission time is short.

I. INTRODUCTION

Powerline communication (PLC) has gained some traction in recent years. Several devices developed to support a high-speed data rate under the standard IEEE Std. 1901, such as HomePlug products, are now on the market. The modulation that is specified in the standard is orthogonal frequency-division multiplexing (OFDM), which is also used in many current wireless communication standards. Besides data rate, energy consumption is also an important performance metric since telecommunications industry has contributed a significant emission of greenhouse gases.

Powerline and wireless channels share some similar characteristics, for example, multipath fading [1]. Some research contribution in wireless communications can be applied for PLC. However, PLC has some distinct factor of its own to be considered such as impulsive noise. To minimize the transmission power in a multiuser wireless OFDM channel, an iterative bit allocation is proposed in [2] and is shown to reduce the required power by a few dB's. The allocation is later applied and simulated in a local area network (LAN) over a PLC channel in [3] and is compared with a suboptimal scheme with less complexity. In [4], the maximum transmission time of all subchannels is minimized for a Zimmermann channel model [5], subject to a total-energy constraint. In addition to

optimizing total energy, power, and transmission time, other parameters that have been adapted are rate, modulation, and a set of subchannels to be used for transmission [2], [6].

In this work, we assume a multipath-fading OFDM channel with single user. We consider a measurement-based model proposed by Zimmermann and Dostert [5] as well as a random model whose temporal channel gains are independent complex Gaussian. In OFDM, data is divided and transmitted onto parallel subchannels. Subject to constraints on total data to be transmitted and transmission time, we would like to find the optimal bit allocation for all subchannels that minimizes total energy consumption.

We propose two bit allocation methods. For a deterministic channel model, we derive the optimal bit allocation over all subchannels that minimize the sum transmission energy. Numerical examples show that the optimal allocation performs significantly better than a uniform allocation in which all subchannels are allotted equal number of bits. The gap between total energy for both methods could be several orders of magnitude. We also propose a threshold-based bit allocation method in which subchannels with ratio between a noise power and squared channel gain larger than a threshold will be allotted the same number of bits and otherwise, subchannels will be allotted zero bit. This on-off allocation is simpler than the optimal allocation if the threshold is known, and can perform close to the optimum when transmission duration is short. For a random channel model, we analyze the optimal threshold, which directly depends on the number of active or 'on' subchannels. A closed-form expression of the total energy consumption per bandwidth with the on-off bit allocation is obtained in a large system limit in which a number of subchannels tend to infinity with fixed total bandwidth.

II. CHANNEL MODEL

A PLC generally consists of electrical cables and circuits designed and installed mainly to deliver electrical power. When it is used to transmit data, a transmitted signal is reflected by impedance mismatching of electric components with different characteristic impedance from many branches of a powerline network. From a transmitter to a receiver, there could be a line-of-sight path as well as several reflected paths. In [5], a channel model that accounts for both multipath propagation and signal attenuation is proposed and the corresponding

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frequency response of an M -path channel is given by

$$H(f) = \sum_{i=1}^M g_i e^{-(a_0 + a_1 f^k) d_i} e^{-j2\pi f \tau_i} \quad (1)$$

where f denotes frequency, g_i is a temporal channel gain for path i , a_0 , a_1 , and k are constants based on cable parameters, d_i denotes cable length for path i , and τ_i denotes delay of path i , which is given by

$$\tau_i = \frac{d_i \sqrt{\varepsilon_r}}{c} = \frac{d_i}{v_P} \quad (2)$$

where ε_r denotes the dielectric constant of the insulating material, and c is the speed of light. All parameters for the often used 15-path model are listed in [6].

We assume that OFDM with N subchannels is used for signal transmission. For a discrete-time point-to-point channel model, the received symbol for the n th subchannel is given by

$$r_n = \sqrt{P_n} H_n x_n + w_n \quad (3)$$

where x_n is a transmitted symbol with zero mean and unit variance, $H_n \triangleq H(f_n)$ is the frequency response at the n th subchannel whose frequency equals f_n , P_n is a transmission power for the n th subchannel, w_n is an additive Gaussian noise with zero mean and variance $B\eta_n$, where B is a subchannel spacing, and η_n is a noise spectral density at the n th subchannel. The associated capacity of the n th subchannel is given by

$$C_n = \log_2 \left(1 + \frac{J_n |H_n|^2}{BT\eta_n} \right) \quad (4)$$

where T denotes a transmission duration and $J_n = P_n T$ is the energy used to transmit data during that duration. The capacity shown in (4) gives the maximum achievable rate. Hence, the maximum number of bits transmitted via subchannel n with bandwidth B is given by

$$Q_n = C_n BT. \quad (5)$$

Substituting (5) into (4), we obtain the energy that is required to transmit Q_n bits and is denoted by

$$J_n = (2^{\frac{Q_n}{TB}} - 1) \frac{BT\eta_n}{|H_n|^2} \quad (6)$$

and the total energy required for all subchannels is given by

$$J \triangleq \sum_{n=1}^N J_n = \sum_{n=1}^N (2^{\frac{Q_n}{TB}} - 1) \frac{BT\eta_n}{|H_n|^2}. \quad (7)$$

Our goal here is not to maximize the capacity, but to minimize the total energy used for transmitting a total of Q bits. In other words, we would like to find the optimal set of number of bits transmitted by each subchannel that minimizes

the total energy as follows

$$\begin{aligned} & \underset{\{Q_n\}}{\text{minimize}} \quad J = \sum_{n=1}^N J_n \\ & \text{subject to} \quad \sum_{n=1}^N Q_n \geq Q, \\ & \quad \quad \quad Q_n \geq 0, \text{ for } 1 \leq n \leq N. \end{aligned} \quad (8)$$

III. PROPOSED BIT ALLOCATIONS FOR DETERMINISTIC CHANNELS

A. Optimal Bit Allocation

With given sets $\{|H_n|^2\}$ and $\{\eta_n\}$, and other system parameters, we form a modified Lagrangian of the considered minimization problem as follows,

$$\mathcal{L} = \sum_{n=1}^N (2^{\frac{Q_n}{TB}} - 1) \frac{BT\eta_n}{|H_n|^2} + \lambda \left(\sum_{n=1}^N Q_n - Q \right) \quad (9)$$

where λ is a Lagrange multiplier. The associated Kuhn-Tucker conditions for the modified Lagrangian are the following [7].

$$Q_n \cdot \left(2^{\frac{Q_n}{TB}} \frac{(\ln 2)\eta_n}{|H_n|^2} - \lambda \right) = 0, \quad \forall n, \quad (10)$$

$$\left(2^{\frac{Q_n}{TB}} \frac{(\ln 2)\eta_n}{|H_n|^2} - \lambda \right) \leq 0, \quad \forall n, \quad (11)$$

$$\lambda \left(\sum_{n=1}^N Q_n - Q \right) = 0, \quad (12)$$

$$\sum_{n=1}^N Q_n - Q \geq 0, \quad (13)$$

$$Q_n \geq 0, \quad \forall n, \quad (14)$$

$$\lambda \geq 0. \quad (15)$$

Based on these conditions, we obtain the optimal solutions by applying Algorithm 1. First, all ratios between the noise power and background noise and squared channel gain $\frac{\eta_n}{|H_n|^2}$, $1 \leq n \leq N$ are ordered in a descending order. We denote the ordered set by $\{G_{(1)}, \dots, G_{(N)}\}$ where $G_{(1)} \geq G_{(2)} \geq \dots \geq G_{(N-1)} \geq G_{(N)}$. Thus, $G_{(1)} = \max_n(\frac{\eta_n}{|H_n|^2})$ and $G_{(N)} = \min_n(\frac{\eta_n}{|H_n|^2})$. We denote a set of the number of bits transmitted on the ordered subchannels by $\{Q_{(n)}\}$ and the number of active subchannels by N_{on} . Algorithm 1 returns the optimal bit allocation for the ordered subchannels $\{Q_{(n)}\}$ and the minimum energy.

Given the total bits Q and $\{G_{(n)}\}$, some subchannels with ratio $\frac{\eta_n}{|H_n|^2}$ larger than certain threshold will not be used to transmit any data. For those subchannels that are active, the amount of data allocated $Q_{(n)}$ will be linearly proportional to $TB \log_2(\frac{G_1}{G_{(n)}})$.

B. On-Off Bit Allocation

Here we propose a suboptimal bit allocation. For a given threshold μ , any subchannel whose ratio $\frac{\eta_n}{|H_n|^2}$ is less than μ , will be allocated equal data and will be active or ‘on’ during the transmission. Otherwise, a subchannel will be allocated

Algorithm 1 Optimal bit allocation

```
1: Input:  $Q, T, B, \{G_{(n)}\}$ 
2: Initial parameters:  $N_{\text{on}} = 0$  and  $Q_{(n)} = 0, \forall n$ 
3: repeat
4:    $J = 0$ 
5:   for  $m = 1 : N_{\text{on}}$  do
6:      $Q_{(m)} = Q_{(1)} - TB \log_2(\frac{G_{(1)}}{G_{(m)}})$ 
7:      $J \leftarrow J + (2^{\frac{Q_{(m)}}{TB}} - 1)(TBG_{(m)})$ 
8:   end for
9:    $N_{\text{on}} \leftarrow N_{\text{on}} + 1$ 
10:   $S = \sum_{n=1}^{N_{\text{on}}} TB \log_2(\frac{G_{(1)}}{G_{(n)}})$ 
11:   $Q_{(1)} = \frac{Q-S}{N_{\text{on}}}$ 
12: until  $|TB \log_2(\frac{G_{(1)}}{G_{(N_{\text{on}})}})| > Q_{(1)}$ 
13:  $N_{\text{on}} \leftarrow N_{\text{on}} - 1$ 
14: return  $\{Q_{(n)}\}$  and  $J$ 
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zero bit and will be inactive or ‘off’. The total energy will depend on the number of ‘on’ subchannels or the threshold μ . For a given set $\{\frac{\eta_n}{|H_n|^2}\}$, the optimal threshold that minimizes the total energy can be found by numerical search. This allocation is less complex than the optimal one and is detailed in Algorithm 2.

Algorithm 2 On-off bit allocation

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1: Input:  $Q, T, B, \{G_{(n)}\}$ 
2: Initial parameters:  $N_{\text{on}} = 1$  and  $Q_{(n)} = 0, \forall n$ 
3:  $J = (2^{\frac{Q}{TB}} - 1)(TBG_{(1)})$ 
4: repeat
5:    $J_{\text{old}} \leftarrow J$ 
6:    $N_{\text{on}} \leftarrow N_{\text{on}} + 1$ 
7:    $J = \sum_{n=1}^{N_{\text{on}}} (2^{\frac{Q}{TB N_{\text{on}}}} - 1)(TBG_{(n)})$ 
8: until  $J > J_{\text{old}}$ 
9:  $N_{\text{on}} \leftarrow N_{\text{on}} - 1$ 
10:  $J \leftarrow J_{\text{old}}$ 
11: return  $\{Q_{(n)}\}$  and  $J$ 
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IV. ANALYSIS OF THE ON-OFF BIT ALLOCATION FOR RANDOM CHANNELS

In this section, we consider a random model of a point-to-point discrete-time OFDM channel. We assume that the received signal is coming from sufficiently many branches of network and hence, the signal propagates through a rich scattering channel. Thus, we model the temporal channel gain g_i in (1) as an independent complex Gaussian random variable with zero mean. Assuming a uniform power-delay profile, g_i ’s have the same variance. In addition, we assume that all paths have comparable lengths and hence, attenuation affects all paths equally. Therefore, for this model, we omit attenuation factors in (1). The noise is assumed to be white with constant spectral density N_0 .

We would like to analyze the total energy resulting from the on-off threshold-based bit allocation proposed in the previous

section. Suppose $\{A_{(n)}\}$ be the ordered set of squared channel gains $|H_n|^2$ in descending order. Applying the threshold μ , the number of active subchannels N_{on} is random and that $A_{(N_{\text{on}})} \geq \mu \geq A_{(N_{\text{on}}+1)}$. With a fixed total bandwidth denoted by $W \triangleq NB$, the expected total energy per bandwidth associated with this allocation is given by

$$E[\bar{J}] = E\left[\frac{J}{W}\right] = E\left[(2^{\frac{Q}{TB N_{\text{on}}}} - 1)TN_0 \frac{1}{N} \sum_{n=1}^{N_{\text{on}}} \frac{1}{A_{(n)}}\right]. \quad (16)$$

Since the joint pdf for $\{A_{(n)}\}$ is complex, evaluating (16) for finite N is not tractable and may not give us much insight. In practice, N is usually large. Hence, we choose to analyze the performance in a large system limit where $N \rightarrow \infty$. We also suppose that the number of ‘on’ subchannels N_{on} scales linearly with N , and denote a fraction of the ‘on’ subchannels by $\bar{N}_{\text{on}} \triangleq N_{\text{on}}/N$. With fixed total bandwidth, B tends to zero as N tends to infinity. In a large system limit mentioned, the energy consumed per bandwidth converges to

$$\bar{J}^\infty = \lim_{\substack{N, N_{\text{on}} \rightarrow \infty \\ B \rightarrow 0}} E\left[\frac{J}{W}\right] \quad (17)$$

$$= (2^{\frac{Q}{TW \bar{N}_{\text{on}}}} - 1)TN_0 \int_{\mu}^{\infty} \frac{1}{x} f_A^\infty(x) dx. \quad (18)$$

where $f_A^\infty(\cdot)$ is an empirical density function of squared channel gain $A_{(n)}$ for all n in the large system limit. Given the threshold μ , $\bar{N}_{\text{on}}$ can be solved from the following equation

$$\bar{N}_{\text{on}} = \int_{\mu}^{\infty} f_A^\infty(x) dx. \quad (19)$$

By letting $F_A^\infty(\cdot)$ be the corresponding empirical distribution function for $A_{(n)}$, the threshold is explicitly shown as an inverse of F_A^∞ as follows

$$\mu = (F_A^\infty)^{-1}(1 - \bar{N}_{\text{on}}). \quad (20)$$

It can be shown that with the assumed channel model described earlier, the limiting empirical density as $N \rightarrow \infty$, is an exponential density function given by

$$f_A^\infty(x) = \frac{1}{\sigma_g^2} e^{-\frac{x}{\sigma_g^2}}, \quad (21)$$

where $\sigma_g^2 = \sum_{i=1}^M E|g_i|^2$ is the sum of all variances of temporal channel gains. The corresponding limiting empirical distribution is given by

$$F_A^\infty(x) = 1 - e^{-\frac{x}{\sigma_g^2}}. \quad (22)$$

Using this distribution function in (20), we obtain the threshold as follows

$$\mu = -\sigma_g^2 \ln \bar{N}_{\text{on}}. \quad (23)$$

Substituting (23) and (21) into (18), we have

$$\bar{J}^\infty = (2^{\frac{Q}{TW \bar{N}_{\text{on}}}} - 1)TN_0 \int_{-\sigma_g^2 \ln \bar{N}_{\text{on}}}^{\infty} \frac{1}{x \sigma_g^2} e^{-\frac{x}{\sigma_g^2}} dx \quad (24)$$

$$= (2^{\frac{Q}{TW \bar{N}_{\text{on}}}} - 1) \frac{TN_0}{\sigma_g^2} E_1(-\ln \bar{N}_{\text{on}}) \quad (25)$$

where an exponential integral is defined by $E_1(x) = \int_x^\infty \frac{1}{t} e^{-t} dt$.

We would like to find the optimal $\bar{N}_{\text{on}}$ that minimize the total transmission energy as follows

$$\bar{N}_{\text{on}}^* = \arg \min_{0 < \bar{N}_{\text{on}} \leq 1} \bar{J}^\infty. \quad (26)$$

The solution to this problem is not in a closed form due to the exponential integral. However, the problem can be easily solved by a numerical method.

V. NUMERICAL RESULTS

In Fig. 1, we consider PLC with the number of subchannels $N = 1024$, subchannel spacing $B = 19.043$ kHz, the target data rate ($R \triangleq Q/T$) of 100 Mbps and noise spectral density used in [8]. A frequency response of the channel is given by (1) with $M = 15$ and all channel parameters found in [6]. We obtain the optimal bit allocation for all subchannels from Algorithm 1 and also find the on-off bit allocation from Algorithm 2. The associated total energy for the two proposed methods is compared with the total energy from a uniform bit allocation in which data is allocated equally on all subchannels. For all allocations, energy consumption increases with increasing data amount. We observe that the optimal allocation performs significantly better than the uniform and on-off bit allocations by approximately 80% and 70%, respectively.

Besides the background noise, the transmitted signal in PLC is also corrupted by an impulsive noise. Impulsive noise is caused by rectifier diodes or by connecting and disconnecting electrical devices. A inter-arrival time between occurrences of impulsive noise can be modeled by an exponential distribution while an impulse duration can be modeled with a two-term Gaussian model. The model of impulsive noise is detailed in [8].

In addition to the background noise, we also take into account an impulsive noise with the worst-case power spectral density, i.e., -84 dBV²/Hz with the mean impulse duration of $4.7 \mu\text{s}$ and inter-arrival time of 10.1 ms. During an occurrence of impulsive noise, the noise spectral density will be increased as indicated. We show the total energy of the three methods affected by an additional impulsive noise in Fig 1 by different markers. We note that the increase in total energy caused by impulsive noise in all three method is very small.

Fig. 2 shows the total energy consumption of all allocations with different transmission-time requirement. Simulation results obtained for this figure also comes from Zimmerman's channel model [5]. As expected, the total energy decreases as transmission time T increases for given data amount (e.g., $Q = 100$ Mb). However, data rate R is also reduced. We observe a larger gap between the uniform and the other two allocations when T is increased. At $T = 10$ s or $R = 10$ Mbps, the gap is as large as an order of magnitude. We also note that the on-off allocation performs close to the optimal allocation especially with a small T .

In Fig. 3, we show a percentage of the number of active or 'on' subchannels over total number of subchannels for

Fig. 1. Total energy consumption of two allocations and the uniform allocation is shown with total transmitted data with $N = 1024$, $B = 19.043$ kHz, and $R = 100$ Mbps.

Fig. 2. The total energy consumption of all allocations is shown with different transmission time and $Q = 100$ Mb.

all three allocations with different data rate R . The same channel model and parameters are applied as in Figs. 1 and 2. The uniform allocation uses all subchannels (100%) for all data rates while the on-off and optimal allocations use fewer subchannels when the required data rate is not high. Only subchannels whose ratio $\frac{\eta_n}{|H_n|^2}$ are below the threshold will be activated to save energy consumption. With $R = 50$ Mbps, the optimal allocation has almost 80% of all subchannels activated while the on-off allocation has 70% activated. As the required data rate is increasing with fixed transmission time, we see that eventually all subchannels are activated to transmit data.

Next we consider a random channel discussed in Section IV with the number of subchannels $N = 1024$, noise spectral density $N_0 = -92$ dBV²/Hz, total bandwidth $W = 19.5$ MHz, and $\sigma_g^2 = 1$. In Fig. 4, we plot the limiting energy per bandwidth $\bar{J}^\infty$ versus $\bar{N}_{\text{on}}$ obtained by (25) for different

Fig. 3. Shown is a percentage of active or 'on' subchannels of all allocations with varying data rates

amount of data Q and transmission time T . We observe that there exists the minimum $\bar{J}^\infty$ and the optimal $\bar{N}_{on}^*$ for each plot, however, the minimum point is not pronounced. For $Q = 150$ Mb and $T = 1$ s, the optimal number of subchannels that should be on is approximately 92%. For lower R , we see smaller $\bar{N}_{on}^*$. According to the figure, for $R = 100$ Mbps, $\bar{N}_{on}^*$ is approximately 0.87 or 890 subchannels should be activated.

Fig. 4. Shown is the total energy per system bandwidth $\bar{J}^\infty$ versus $\bar{N}_{on}$ for $W = 19.5$ MHz, $N_0 = -92$ dBV²/Hz, and $\sigma_g^2 = 1$.

In the last figure, we show the optimal $\bar{N}_{on}$ that minimizes the total energy with different total transmitted data Q with the same set of system parameters as Fig. 4. We note that the fraction of subchannels to be activated increases as amount of data to be transmitted increases or transmission time decreases.

Fig. 5. The optimal $\bar{N}_{on}^*$ is shown with total transmitted data Q for different transmission time.

VI. CONCLUSIONS

The optimal and on-off bit allocations can significantly reduce the total energy when compared to the uniform bit allocation. When the transmission time is decreased, the numerical results show that the performance of both the optimal and on-off allocation is similar. The number of active subchannels for the two proposed allocations increases with data rate for a fixed transmission time. We have analyzed the total energy associated with the on-off bit allocation and is able to obtain the optimal number of 'on' subchannels. This parameter should be a useful guideline for system designers or engineers.

In this work, only uniform power-delay profile is considered for a random model and the current analysis only takes into account the background noise. Our future work may include performance analysis for other delay profiles and incorporating impulsive noise.

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