Optimal subcarrier allocation for 2-user downlink multiantenna OFDMA channels with beamforming interpolation

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Abstract—**We consider a 2-user multiantenna downlink OFDMA with limited feedback. Assuming perfect channel state information (CSI) at mobile stations, each mobile equipped with a single receive antenna quantizes and feeds back to a base station one optimal transmit beamforming vector for each subcarrier cluster. A base station then applies constant or linear interpolation methods to obtain all other beamforming vectors for each cluster. For given feedback rate and channel condition, we analyze the optimal cluster size, the number of clusters, and thus, subcarrier allocation for the two users. Numerical examples show that the optimized sum capacity can be significantly higher than a sum capacity with arbitrary system parameters.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) has been used in many current air-interface standards for wireless systems. In OFDM, a frequency-selective fading wideband channel is converted into multiple flat-fading subchannels or subcarriers, which can greatly simplify a receiver. In orthogonal frequency-division multiple access (OFDMA), different users operate in different nonoverlapping OFDM subcarriers and thus, avoid interfering with other users. OFDMA is combined with multiple antennas at either the transmitter and/or receiver to increase spectral efficiency in IEEE 802.16 and LTE-advanced standards [1].

For a wireless OFDMA channel, there has been a significant interest on how to utilize or allocate available transmission power, a number of subcarriers per user, and a number of feedback bits more efficiently. In [2], optimal subcarrier and power allocation are based on water-filling an inverse of channel spectrum. Reference [3] has proposed to allocate subcarriers for each user by maximizing its channel capacity while [4] has proposed to allocate a feedback rate for each user by maximizing sum capacity. However, all of these cited work has assumed a single transmit and receive antenna while our work considers a transmitter with multiple antennas.

For a multiantenna system, transmit beamforming is a simple technique to increase an achievable rate by adjusting transmit-antenna coefficients in the direction of the strongest channel mode [5]. However, transmit beamforming requires current channel state information (CSI) at the transmitter. CSI can be estimated at the receiver by pilot signal. But, the transmitter can not estimate CSI directly, especially in frequency division duplex (FDD). Hence, CSI needs to be quantized and fed back from receiver to transmitter via a rate-limited feedback channel. Many quantization schemes have been proposed and analyzed for both point-to-point and multiuser channels [6], [7, see references therein].

Our work is an extension of [8], [9] in which we consider a single-user OFDM with multiple antennas. Here we consider a 2-user downlink OFDMA with multiple transmit and single receive antennas, and assume a fixed total feedback bits and an equal power allocation among the two users. To reduce a number of feedback bits required to quantize all transmit beamforming vectors, we choose to quantize only a few selected beamforming vectors and the rest will be interpolated from selected ones. A beamforming vector is quantized at a mobile or receiver and is then fed back to the base station or transmitter. A constant interpolation is first applied and was proposed by [10] in which all subcarriers in a cluster use the same quantized beamforming vector. We apply random vector quantization [6] for quantizing beamforming vectors and derive an approximation for the sum-capacity upper bound. The second interpolation we consider is a linear interpolation, which was proposed by [11] and later modified by us [9]. Linearly interpolating transmit beamforming is more complex than constant interpolating, but gives substantial increase in the performance. With capacity analysis of the constant interpolation method, we show that the optimal subcarrier allocation depends on feedback allocation for user 1 and 2 as well as frequency selectivity of channels of the two users.

II. Channel Model

We consider a discrete-time downlink 2-user multiantenna OFDMA channel as shown in Fig. 1. A base station is equipped with N_t transmit antennas while user 1 and user 2 are each equipped with a single antenna. We assume that the base station transmits an OFDM signal through N total subcarriers

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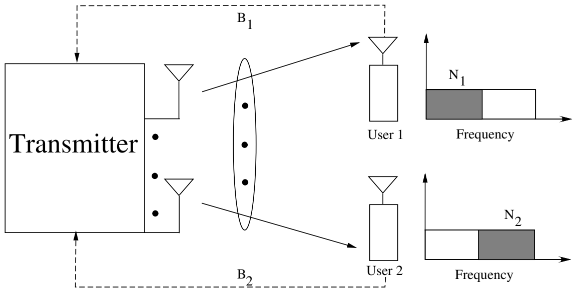

Fig. 1. Shown is a 2-user multiantenna OFDMA channel.

to the two users over nonoverlapping subcarriers. Without loss of generality, we assume that user 1 is allocated the first N_1 subcarriers while user 2 is allocated the rest $N_2 = N - N_1$ subcarriers as shown in Fig. 1. Furthermore, we assume that the transmitted signal from the base station transverses to user 1 and user 2 through L_1 and L_2 paths, respectively. Applying a discrete Fourier transform, the frequency response for user 1 from the n_tth transmit antenna for the nth subcarrier is given by

$$H_{n,n_t;1} = \sum_{l=1}^{L_1} h_{l,n_t;1} \mathrm{e}^{-\frac{j2\pi(l-1)(n-1)}{N}} \tag{1}$$

where $h_{l,n_t;1}$ is the channel gain for the lth path between the n_tth transmit antenna and user 1's antenna. Assuming a rich scattering and sufficiently large distance between transmit antennas, $h_{l,n_t;1}$ for all L_1 paths are independent complex Gaussian distributed with zero mean and variance $\frac{1}{L_1}$. Let $\boldsymbol{H}_{n;1}$ denote an $N_t \times 1$ channel vector for user 1 at the nth subcarrier, whose entry is $H_{n,n_t;1}$ shown in (1). Thus,

$$\boldsymbol{H}_{n;1} = [H_{n,1;1} \quad H_{n,2;1} \quad \cdots \quad H_{n,N_t;1}]^T. \tag{2}$$

For user 2, a random variable $h_{l,n_t;2}$ and an $N_t \times 1$ random vector $\boldsymbol{H}_{n;2}$ can be similarly defined.

Assuming a transmit beamforming or a rank-one precoding, the received signal on the nth subcarrier for user 1 is given by

$$r_{n;1} = \boldsymbol{H}_{n;1}^{\dagger} \boldsymbol{v}_{n;1} x_{n;1} + z_{n;1}, \quad 1 \le n \le N_1 \tag{3}$$

where $\boldsymbol{v}_{n;1}$ is an $N_t \times 1$ unit-norm beamforming vector for user 1, $x_{n;1}$ is a transmitted symbol for user 1 with zero mean and unit variance, and $z_{n;1}$ is an additive white Gaussian noise with zero mean and variance σ_z^2. Thus, the sum capacity for user 1 over N_1 subcarriers is given by

$$C_1 = \sum_{n=1}^{N_1} E\left[\log(1 + \rho|\boldsymbol{H}_{n;1}^{\dagger} \boldsymbol{v}_{n;1}|^2)\right] \tag{4}$$

where the expectation is over distribution of $\boldsymbol{H}_{n;1}$. We assume a uniform power allocation for all subcarriers and hence, the background signal-to-noise ratio (SNR) for each subcarrier $\rho = 1/\sigma_z^2$. A sum capacity for user 2 over N_2 subcarriers is similar to that for user 1 and is given by

$$C_2 = \sum_{n=N_1+1}^{N} E\left[\log(1 + \rho|\boldsymbol{H}_{n;2}^{\dagger} \boldsymbol{v}_{n;2}|^2)\right]. \tag{5}$$

Since subcarriers of the two users do not overlap, there is no interference among users. The system capacity is given by

$$C = C_1 + C_2. \tag{6}$$

From (4) and (5), we note that the capacity is a function of two sets of transmit beamforming vectors $\{\boldsymbol{v}_{n;1}\}$ and $\{\boldsymbol{v}_{n;2}\}$. A user or mobile with perfect channel information can optimize the sum capacity over transmit beamforming vectors and send the optimal beamforming vectors to the base station or transmitter via a feedback channel. Since the feedback channel between the receiver and the transmitter has finite rate, quantization of the optimized beamforming vector is required. In this study we apply a random vector quantization (RVQ) codebook whose entries are independent isotropically distributed vectors to quantize a transmit beamforming vector. RVQ is simple, but was shown to perform close to the optimum codebook [6], [12].

We assume that feedback rates available for user 1 and user 2 are B_1 and B_2 bits per update, respectively, and the total feedback is $B = B_1 + B_2$. For user 1, there are N_1 beamforming vectors (one for each subcarrier) to quantize. For an equal bit allocation per subcarrier, each beamforming vector is quantized with B_1/N_1 bits. Let us denote the RVQ codebook for user 1 by $\mathcal{V}_1 = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \ldots, \boldsymbol{w}_{2^{B_1/N_1}}\}$ where there are $2^{B_1/N_1}$ entries in the codebook. User 1 selects for the nth subcarrier

$$\hat{\boldsymbol{v}}_{n;1} = \arg\max_{\boldsymbol{w} \in \mathcal{V}_1} \log(1 + \rho|\boldsymbol{H}_{n;1}^{\dagger} \boldsymbol{w}|^2) \tag{7}$$

$$= \arg\max_{\boldsymbol{w} \in \mathcal{V}_1} |\boldsymbol{H}_{n;1}^{\dagger} \boldsymbol{w}|^2 \tag{8}$$

that maximizes an instantaneous achievable rate and the associated rate for the nth subcarrier is bounded by

$$E\log(1 + \rho|\boldsymbol{H}_{n;1}^{\dagger} \hat{\boldsymbol{v}}_{n;1}|^2) \le \log(1 + \rho E|\boldsymbol{H}_{n;1}^{\dagger} \hat{\boldsymbol{v}}_{n;1}|^2) \tag{9}$$

where Jensen's inequality is applied. It has been shown by [12] that $\|\boldsymbol{H}_{n;1}\|^2$ and $|\boldsymbol{H}_{n;1}^{\dagger} \hat{\boldsymbol{v}}_{n;1}/\|\boldsymbol{H}_{n;1}\||^2$ are independent random variables. It can be easily shown that $E\|\boldsymbol{H}_{n;1}\|^2 = N_t$ while $E|\boldsymbol{H}_{n;1}^{\dagger} \hat{\boldsymbol{v}}_{n;1}/\|\boldsymbol{H}_{n;1}\||^2$ was analyzed in [12]. Combining these expectations with (4) and (9), we obtain the upper bound on the sum capacity for user 1 as follows

$$C_1 \le N_1 \log(1 + \rho N_t \gamma(\frac{B_1}{N_1})) \tag{10}$$

where $\gamma(x) = 1 - 2^x \beta\left(2^x, \frac{N_t}{N_t - 1}\right)$. $\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ is the beta function and the gamma function $\Gamma(x) = \int_0^{\infty} t^x \mathrm{e}^{-t}\, \mathrm{d}t$. The capacity upper bound for user 2 can be similarly obtained. Thus, the system capacity with this equal bit-per-subcarrier allocation is upper bounded by

$$C \le N_1 \log(1 + \rho N_t \gamma(\frac{B_1}{N_1})) + N_2 \log(1 + \rho N_t \gamma(\frac{B_2}{N_2})). \tag{11}$$

We note that the capacity bound depends on the number of feedback bits per subcarrier allocated for both users (B_1/N_1 and B_2/N_2). These ratios could be small due to a large number of subcarriers in a practical OFDM system. Hence, this may result in a large quantization error, which leads to a substantial performance loss.

III. Optimizing subcarrier allocation with beamforming interpolation

Feeding back transmit beamforming vectors of all subcarriers requires quantizing NN_t complex coefficients and thus, a large number of feedback bits. We note that adjacent subcarriers in OFDM are highly correlated since the number of channel taps is much lower than that of subcarriers ($L_1, L_2 \ll N$). The optimal transmit beamformers, which depend on channel matrices, are also highly correlated as well. In this section, we apply two interpolation methods to reduce the number of feedback bits while maintaining the performance.

A. Constant Interpolation

In the first method, we group adjacent subcarriers into a cluster and use the same quantized beamforming vector for all subcarriers in the cluster. We denote a number of subcarriers in one cluster for user 1 by M_1. Thus, the number of subcarrier clusters for user 1 is given by $K_1 \triangleq \lfloor N_1/M_1 \rfloor$ and there might be a few remaining subcarriers. The number of feedback bits allocated for each cluster for user 1 is equal to B_1/K_1. All B_1/K_1 bits are used to quantize the beamforming vector of the centered subcarrier for odd M_1 and one subcarrier off the center for even M_1. The beamforming vector used for the kth cluster is given by

$$\hat{\boldsymbol{v}}_{kM_1+m;1} = \begin{cases} \arg\max_{\boldsymbol{w}\in\mathcal{V}_1} \ |\boldsymbol{H}^\dagger_{kM_1+\frac{M_1+1}{2};1}\boldsymbol{w}|^2 & \text{for odd } M_1 \\ \arg\max_{\boldsymbol{w}\in\mathcal{V}_1} \ |\boldsymbol{H}^\dagger_{kM_1+\frac{M_1}{2};1}\boldsymbol{w}|^2 & \text{for even } M_1 \end{cases} \tag{12}$$

where $1 \le m \le M_1$ and $0 \le k \le K_1 - 1$.

If N_1/M_1 is not integer, there exists some remaining subcarriers, which do not belong in a cluster. We propose to set the transmit beamforming for these subcarriers to be that of the last cluster as follows

$$\hat{\boldsymbol{v}}_{K_1M_1+q;1} = \hat{\boldsymbol{v}}_{K_1M_1;1} \quad \text{for } 1 \le q \le N_1 - K_1M_1 \tag{13}$$

To analyze the sum capacity of this method, we apply the following approximation, which was shown in [8],

$$E|\boldsymbol{H}^\dagger_{p+q;1}\hat{\boldsymbol{v}}_{p;1}|^2 \approx \gamma(\frac{B_1}{K_1})[1 + \frac{N_t}{L_1^2}\varphi^2(q)] \tag{14}$$

where

$$\varphi(x) = \frac{\sin\frac{L_1\pi x}{N}}{\sin\frac{\pi x}{N}}. \tag{15}$$

As q increases or as the quantized beamformer moves further away from the centered subcarrier, the received power or $|\boldsymbol{H}^\dagger_{p+q;1}\hat{\boldsymbol{v}}_{p;1}|^2$ is degrading.

With the approximation (14) and Jensen's inequality, we obtain the approximate upper bound for the sum capacity of user 1 for odd M as follows

$$\begin{aligned} \tilde{C}_1 &= K_1 \log(1 + \rho N_t \gamma(\frac{B_1}{K_1})) \\ &\quad + 2K_1 \sum_{q=1}^{\frac{M_1-1}{2}} \log(1 + \rho\gamma(\frac{B_1}{K_1})[\frac{N_t}{L_1^2}\varphi^2(q) + 1]) \\ &\quad + \sum_{r=1}^{N_1-K_1M_1} \log(1 + \rho\gamma(\frac{B_1}{K_1})[\frac{N_t}{L_1^2}\varphi^2(r + \frac{M_1}{2}) + 1]). \end{aligned} \tag{16}$$

where the last sum in (16) accounts for the remaining subcarriers that do not belong to any cluster. For even M,

$$\begin{aligned} \tilde{C}_1 &= K_1 \log(1 + \rho N_t \gamma(\frac{B_1}{K_1})) \\ &\quad + 2K_1 \sum_{q=1}^{\frac{M}{2}-1} \log(1 + \rho\gamma(\frac{B_1}{K_1})[\frac{N_t}{L_1^2}\varphi^2(q) + 1]) \\ &\quad + K_1 \log(1 + \rho\gamma(\frac{B_1}{K_1})[\frac{N_t}{L_1^2}\varphi^2(\frac{M}{2}) + 1]) \\ &\quad + \sum_{r=1}^{N_1-K_1M_1} \log(1 + \rho\gamma(\frac{B_1}{K_1})[\frac{N_t}{L_1^2}\varphi^2(r + \frac{M_1}{2}) + 1]). \end{aligned} \tag{17}$$

The approximate upper bound for sum capacity of user 2 is similar to (16) and (17) where we replace $\{B_1, K_1, M_1, N_1, L_1\}$ with $\{B_2, K_2, M_2, N_2, L_2\}$.

Given feedback rates and other channel parameters, we would like to determine subcarrier allocation N_1 and N_2, which maximize the system sum rate. For a constant interpolation, we can obtain subcarrier allocation that maximizes the approximate upper bound of the sum capacity as follows

$$N_1^* = \arg \max_{\substack{1 \le N_1 \le N-1 \\ N_1+N_2=N}} \tilde{C}_1 + \tilde{C}_2 \tag{18}$$

and $N_2^* = N - N_1^*$. Solving (18) requires either integer optimization for which there exist many available tools or exhaustive search.

B. Linear Interpolation

To increase the performance, we apply a more sophisticated linear interpolation proposed by [11] and later improved by [9]. Similar to the constant interpolation, all subcarriers are grouped into K_1 clusters for user 1 and K_2 clusters for user 2 and each cluster for user 1 and user 2 consists of M_1 and M_2 contiguous subcarriers, respectively. In each cluster, the beamforming vector of the first subcarrier is quantized with RVQ. Thus, the number of vectors to be quantized for user 1 is $K_1 + 1$. We note that the one additional vector that requires quantization arises from interpolating the last cluster. Thus, each vector will be quantized with $B_1/(K_1 + 1)$ bits.

Beamforming vectors in a cluster are linear combination of the quantized beamforming vector of the first subcarrier in the cluster and that in the next cluster as follows

$$\hat{\boldsymbol{v}}_{kM_1+m;1} \triangleq \frac{(1 - \frac{m}{M_1})\hat{\boldsymbol{v}}_{kM_1;1} + \frac{m}{M_1}\mathrm{e}^{j\theta_m}\hat{\boldsymbol{v}}_{(k+1)M_1;1}}{\|(1 - \frac{m}{M_1})\hat{\boldsymbol{v}}_{kM_1;1} + \frac{m}{M_1}\mathrm{e}^{j\theta_m}\hat{\boldsymbol{v}}_{(k+1)M_1;1}\|} \tag{19}$$

for $1 \leq m \leq M_1 - 1$ and $0 \leq k \leq K_1 - 1$, where θ_m is a phase-rotation parameter whose expression was derived in [9]. Beamforming interpolation for user 2 can be performed in a similar manner. If exists, any remaining subcarrier that does not belong in a cluster will have its beamforming set to the last RVQ-quantized beamforming $\hat{\boldsymbol{v}}_{K_1 M_1;1}$ for user 1 or $\hat{\boldsymbol{v}}_{K_2 M_2;2}$ for user 2.

Analyzing the sum capacity of this linear interpolation is more complicated than that of constant interpolation and remains an open problem. Thus, the performance of this method will be shown by simulation results.

IV. Numerical Results

To show performance of the proposed constant and linear interpolation methods, Monte Carlo simulation is used with 3000 channel realizations. We also assume that the feedback channel is error- and delay-free. Fig. 2 compares system capacity of equal bit-per-subcarrier, constant interpolation, and linear interpolation methods. In this figure, we apply equal subcarrier allocation and feedback allocation with $N = 64$ and $N_t = 5$. All solid lines show simulation results for the three methods while the only dashed line shows the analytical bound for equal bit-per-subcarrier method in (11). We note that the linear interpolation outperforms constant and equal bit-per-subcarrier methods as expected for any given total feedback. With 80 total feedback bits, system capacity with the linear interpolation method is 50% larger than that with the constant method. For a larger number of feedback bits ($B \geq 100$), we do not have simulation results for the two interpolation methods due to a search complexity of RVQ, which employs exhaustive search. We note that the capacity upper bound of the equal bit-per-subcarrier method predicts the simulation results well when B is large. A performance gap between interpolation methods and the equal bit-per-subcarrier method is large for a low feedback rate and is diminishing for a very high feedback rate.

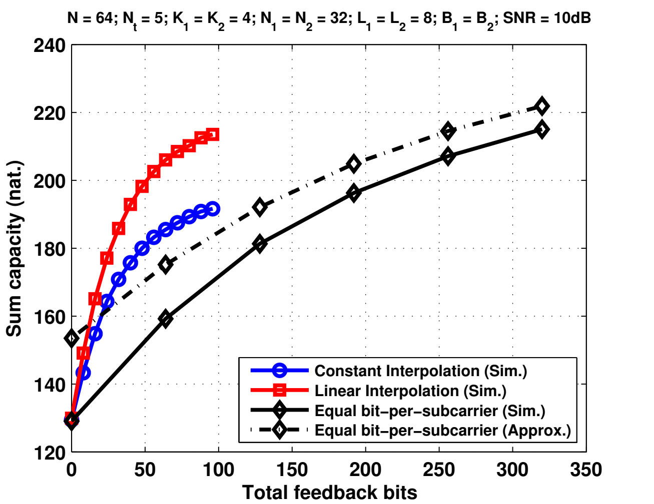

Fig. 2. Sum capacity of equal bit-per-subcarrier, constant, and linear interpolation methods are shown with total feedback bits B for $N = 64$, $K = 4$, $N_1 = N_2 = 32$, $B_1 = B_2$ and $\rho = 10$ dB.

In Fig.3, we plot a sum capacity of the constant interpolation method obtained by numerical simulations and the approximate capacity bound with total feedback bits for different number of channel taps for user 1 (L_1) and fixed $L_2 = 4$. The solid lines show the numerical results while the dashed lines show the approximate upper bound. We observe that the approximate upper bound is about 10% larger than the actual capacity. The discrepancy can be attributed to a large system limit and Jensen's inequality. Both the bound and the capacity increase when more feedback is available for users. In this figure, we observe that as channel for user 1 becomes more frequency selective, the performance decreases. To maintain the performance, the number of clusters K_1 as well as the number of feedback bits B_1 need to be increased.

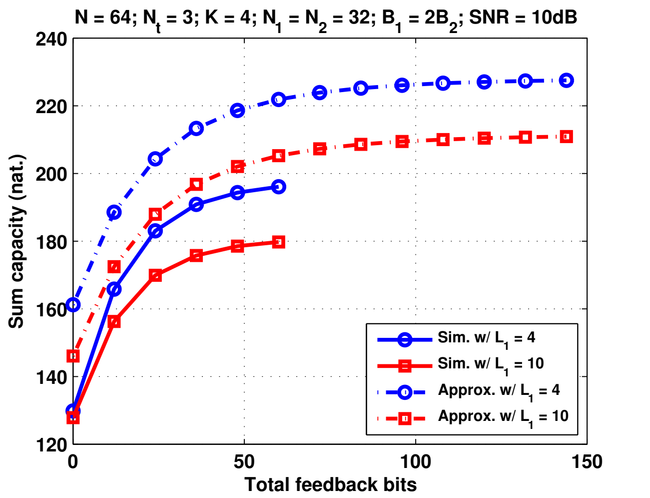

Fig. 3. A sum capacity of the constant interpolation method is shown with a number of total feedback bits B for $N = 64$, $N_t = 3$, $K_1 = K_2 = 4$, $N_1 = N_2 = 32$, $B_1 = 2B_2$, $L_2 = 4$, and $\rho = 10$ dB.

Fig. 4 shows three sets of sum capacity against the number of subcarriers with different L_1 and constant interpolation. The first set is derived from the approximation while the second and third sets are simulation results. The sum capacity of the first and second sets is optimized over the number of clusters (K_1 and K_2) and the number of subcarriers in each cluster (M_1 and M_2). The gap between the approximation and simulation results is not small (less than 20%). However, the approximation derived in Section III can accurately predict the optimal subcarrier allocation for user 1. We note that when user 1 experiences flat fading ($L_1 = 1$), almost all subcarriers should be assigned to user 1. As channel for user 1 becomes more frequency selective, fewer subcarriers should be allocated to user 1. We also display the third set of sum capacity in which $K_1 = K_2 = 4$. Comparing the second and third sets of sum capacity, we see that restricting a number of clusters (or a number of subcarriers in a cluster) can have a detrimental effect on the performance although finding the optimal M_1 and M_2 while fixing K_1 and K_2 is simpler than finding the optimal M_1, M_2, K_1, and K_2.

Fig. 5 and 6 show a ratio of the optimal number of

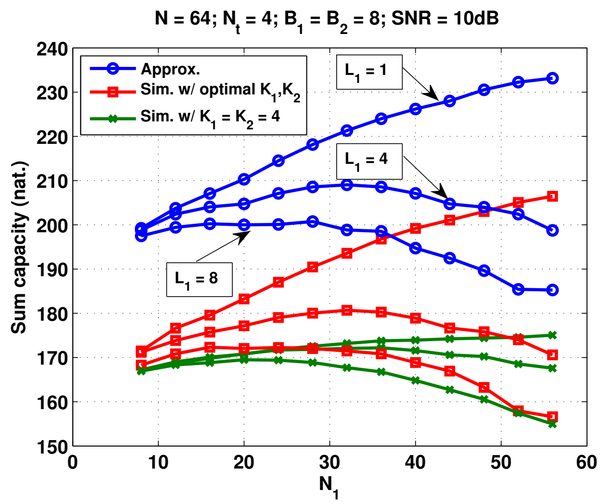

Fig. 4. Sum capacity of 2-user 3×1 OFDMA channel with constant interpolation is shown against N_1 with different L_1, $N = 64$, $K = 4$, $L_2 = 4$, $B_1 = B_2 = 8$, and $\rho = 10$ dB.

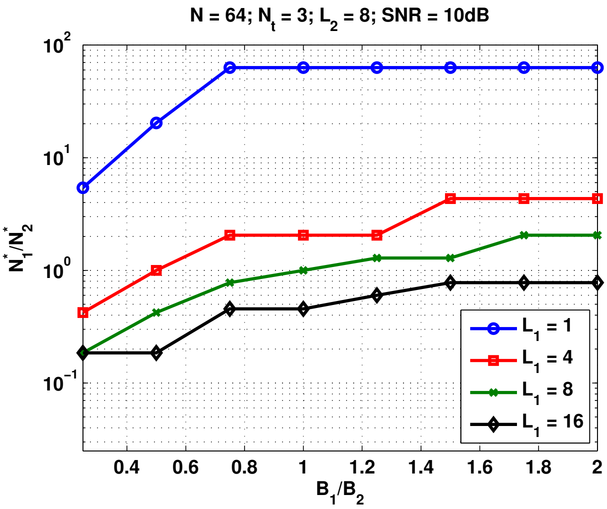

Fig. 6. The optimal N_1^*/N_2^* is shown with B_1/B_2 for $N = 64$, $N_t = 3$, and $\rho = 10$ dB.

subcarriers between user 1 and 2 (N_1^*/N_2^*) obtained from the approximation derived in Section III with L_1/L_2 and B_1/B_2, respectively. In Fig. 5, we observe that N_1^*/N_2^* decreases when L_1/L_2 increases. In other words, when user 1's channel becomes more frequency selective than that of user 2, the number of subcarriers allocated to user 1 should be reduced while the number of subcarriers allocated to user 2 should be increased. We also note that the optimal N_1^*/N_2^* increases with the number of feedback bits allocated to user 1. Furthermore, the difference between the optimal N_1 and N_2 becomes less significant when only a few bits of feedback are available, e.g., $B_1 = B_2 = 8$. In Fig. 6, as we expect, the number of subcarriers allocated to user 1 should increase when more feedback bits are available for user 1.

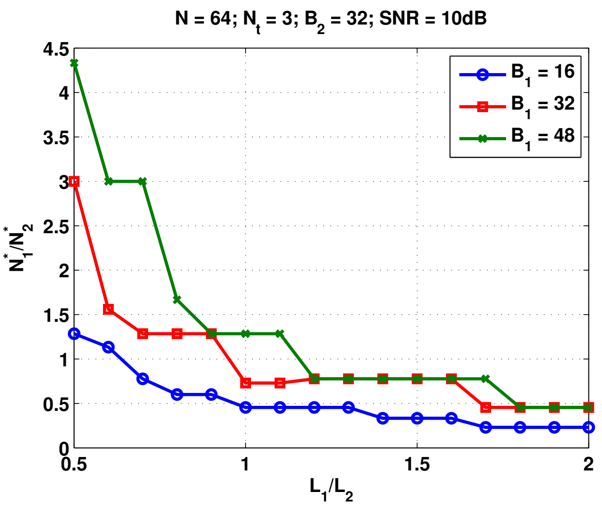

Fig. 5. The optimal N_1^*/N_2^* is shown with L_1/L_2 for $N = 64$, $N_t = 3$, and $\rho = 10$ dB.

In Fig. 7, we show the optimal number of clusters K_1 and K_2, and the optimal number of subcarriers per cluster M_1 and M_2 versus feedback ratio B_1/B_2. The optimal number of clusters for user increases or decreases with available feedback while the cluster size changes very little. The figure gives us the more detailed look at the optimal system parameters.

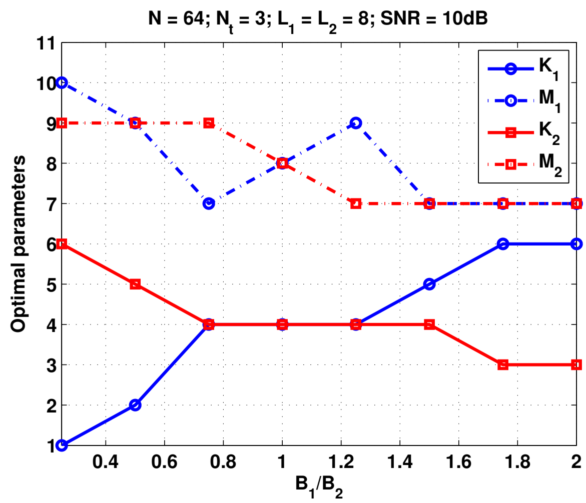

Fig. 7. The optimal K_1, K_2, M_1, and M_2 are shown with B_1/B_2 for $N = 64$, $N_t = 3$, and $\rho = 10$ dB.

V. Conclusions

With transmit beamforming interpolation in a 2-user multiantenna OFDMA channel, we have analyzed the optimal subcarrier allocation for each user that maximizes sum capacity. Through analysis and numerical simulations, we have shown that the optimal subcarrier allocation for user depends on frequency selectivity of user's channel, available feedback rate, and the number of transmit antennas. Specifically, user's subcarrier allocation should decrease with the number of channel paths and increase with feedback rate. In examples shown, operating at the optimal subcarrier allocation can give much higher performance gain than operating at arbitrary parameters does. Here we have only considered 2-user channel and in the future work, we plan to extend the results to OFDMA with an arbitrary number of users.

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