Moiré Techniques

Note#: These briefs document for description my presentation on PowerPoint. You can be downloading my PowerPoint file on my website at http://pirun.ku.ac.th/~fsciwrs/articles/metro/optical/moire_techniques.ppt

Scope

- Introduction to moiré metrology
- Techniques used in moiré metrology
- Applications
- Summary

Introduction

Optical Metrology and Light

Metrology is the science of weights and measure.
Light, electromagnetic periodic perturbation propagation in space at a constant velocity.

Background

17th century:	Robert Hook, who described light as a rapid vibration motion of the medium propagation as a very grate speed.
1629-95:	Christian Huygens, who described light base on modern wave theory.
19th century:	Thomus Young suggested the transverse propagation of light waves.
Laster:	James C. Maxwell established as the electromagnetic field.
1870s:	Albert Abraham Michelson → interferometer, it is the basic of the most important tool in optical metrology.

Moiré Effect

The moiré effect denotes a fringe pattern formed by the superposition of two grid structures of similar period.
Moiré is the French word refers to "watered or wavy appearance".
The term moiré effect refers to a geometrical interference formed when two transmitting screen of similar motif partly overlap.

Analysis of the Moiré Pattern

The gratings $G1$ and $G2$ are described by the two set of equations, respectively

$$y\cos\theta/2 = x\sin\theta + np \qquad n = 0, \pm 1, \pm 2...$$
$$y\cos\theta/2 = -x\sin\theta + mp \qquad m = 0, \pm 1, \pm 2...$$

The geometrical loci of all point intersection of the two gratings form an array of bright fringe whose index is given by

$$l = m - n \qquad l = 0, \pm 1, \pm 2...$$

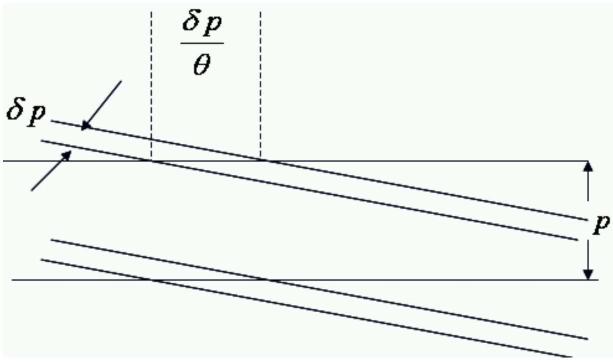
$$l = \frac{2x\sin\theta/2}{p}$$
$$x = \frac{lp}{2\sin\theta/2}$$

For small value of θ ($\sin\theta \approx \theta$, in radians) we obtain

$$x \cong \frac{lp}{\theta}$$

Namely, equation represents a set of straight lines perpendicular to the x axis, whose spacing is θ^{-1} time the pitch the original gratings.

Moiré pattern response of grating's deformations
This fringe analysis, it is an appropriate time to demonstrate how the moiré effect is applied to optical metrology.
Assume one of the gratings is slightly distorted.
Case1: One grating shift δp and parallel, where $\delta p < p$



Fringe shift resulting from translation of one grating

$$y\cos\theta/2 = x\sin\theta/2 + \delta p + np$$
$$y\cos\theta/2 = -x\sin\theta/2 + np$$

The fringe position

$$x = \frac{(m-n)p}{2\sin\theta/2} + \frac{\delta p}{2\sin\theta/2} \cong \frac{lp}{\theta} + \frac{\delta p}{\theta}$$

Namely, the fringes are shifted amount θ^{-1} times the original grating's shift.

Case2: One grating rotated with the optical axis by an angle ϕ

$$y\cos(\theta/2+\phi) = x\sin(\theta/2+\phi) + np$$
$$y\cos\theta/2 = -x\sin\theta/2 + \delta p + np$$

$$x = \frac{lp}{2\sin((\theta+\phi)/2)} \cong \frac{lp}{\theta+\phi}$$

Case3: beats phenomenon

From $I \propto |A|^2$

$$|A|^2 = A_{01}^2 + A_{02}^2 + 2A_{01}A_{02}\cos(\alpha_1 - \alpha_2)$$

$$|A|^2 = A_{01}^2 + A_{02}^2 + 2A_{01}A_{02}\cos 2\pi ct\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

In the corresponding moiré case we combine two parallel gratings

$$y = np_2$$
$$y = mp_1$$

Where $p_1 = kp_2$

$$l = y\left(\frac{1}{p_2} - \frac{1}{p_1}\right)$$

Techniques used in moiré metrology

Fringe quality improvement and readout techniques

Grating's Stripes Multiplication and Addition

$$t_1(x,y) = a + a\cos\left(\frac{2\pi}{p}x\right)$$

$$t_2(x,y) = a + a\cos 2\pi\left(\frac{x}{p} + \psi(x)\right)$$

Stripes Multiplication

$$t(x,y) = t_1 t_2$$

The fringe result from multiplication of the transition functions of two grating and they occur when the two grating are superimposed.
Stripes multiplication reduces the number of fringe with out any loss in the accuracy.

Addition

$$t(x,y) = t_1 + t_2$$

Spatial Filtering

Spatial filtering removes the high spatial frequency and leaves only the moiré pattern

Phase Shift Fringe Readout Method

One of the fringes is move by 1/3 of its pitch.

$$I(x) = A + B\sin(kx + \varphi)$$

Electronic Heterodyne Readout Method
Support phase shift

Information Extraction from Distorted Gratings

Distorted Gratings general form:

$$y + f_i(x, y) = np$$

Where $i = 1, 2, 3..$ $n = 0, \pm 1 \pm 2...$ and $f_i(x, y)$ is the measured quantity

Mapping $f(x, y)$

The simplest moiré mapping is done by superimposing the undistorted grating with the distorted one to obtain an infinite fringe moiré pattern:

$$y + f(x, y) = np \,,\ y = mp$$
$$f(x, y) = lp \quad \text{Where } l = m - n$$

A contour map of $f(x, y)$ incremented by p

Another solution very dense grating

$$2y + f(x, y) = lp \qquad \text{Where } l = n + m$$

Our eye resolves the low density grating
Overlook the high density

Mapping of Differences

Two functions are difference
Two distorted gratings are superimposed to form a moiré pattern:

$$y + f_1(x, y) = np$$
$$y + f_2(x, y) = mp$$
$$f_1(x, y) - f_2(x, y) = lp$$

This result is a contour of the difference between the two function which incremented by p

Mapping the sum

One infinite fringe map obtained in:

$$x - \frac{f_1(x, y)}{\theta} = n\left(\frac{p}{\theta}\right)$$

The second grating is produced by reversing the angle $\theta / 2$ of the original versus the reference grating:

$$x + \frac{f_2(x, y)}{\theta} = l\left(\frac{p}{\theta}\right) \quad l = n - m$$

The sum of the finite fringe is the contour map:

$$f_1(x,y)+f_2(x,y)=mp$$

Multiplication by a Factor
Multiply the function by a factor M
Used the beats phenomenon described

$$y+f(x,y)=np_1$$
$$y=mp_2$$

Where $p_1 \neq p_2$

$$l=m-n=\frac{y+f(x,y)}{p_1}-\frac{y}{p_2}$$

And beats pitch $\frac{1}{p}=\frac{1}{p_1}-\frac{1}{p_2}$

So

$$y+Mf(x,y)=lp \qquad \text{Where } M=p/p_1$$

Weighted Sums and Difference
Two gratings

$$y+f_1(x,y)=np_1$$
$$y+f_2(x,y)=mp_2$$

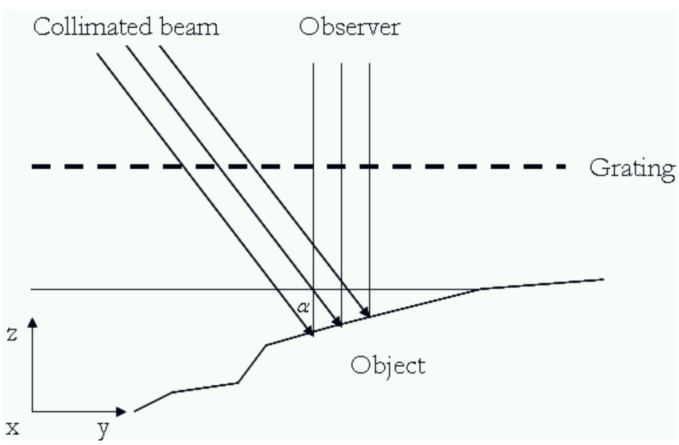
The moiré pattern obtained is

$$y+\frac{p}{p_1}f_1(x,y)-\frac{p}{p_2}f_2(x,y)=lp$$

Applications

Shadow moiré and the Grating Hologram

The basic shadow moiré setup comprises … (see figure)



Shadow moiré setup

Denote $h(x, y)$ is the height variation function.

The amount of distortion is proportion to $h(x, y)\tan\alpha$

$$y = mp, \qquad m = 0, \pm 1, \pm 2...$$
$$y + h(x, y)\tan\alpha = np, \qquad n = 0, \pm 1, \pm 2...$$

Consider

Case1: Straight grating and its deformed shadow. → parallel

Result

- Moiré pattern
- Contour phase constant difference

Case2: $h(x, y)\tan \neq 0$

Result

$$h(x, y)\tan\alpha = (n - m)p$$
$$h(x, y) = lp\cot\alpha$$

Namely, the topographic contour map of the object with a height increment $p\cot\alpha$ between successive fringes

Moiré analysis of Strain

Strain measurement

- The most devices for strain analysis are the strain gauge.
- Should be done by "touching" the measured object.

Here: a grating is glued or etched on a flat test surface and by applying a stress to the surface, the grating will be distorted.

In general, if the surface $f(x, y)$ is transformed to a surface $f(x + \delta x, y + \delta y)$, then the grating which is glued onto the surface

$$\upsilon(x, y) = np \qquad n = 0, \pm 1, \pm 2...$$

With its shifted replica, i.e.,

$$\upsilon(x, y + \delta y) = mp \qquad m = 0, \pm 1, \pm 2...$$

Where $\upsilon(x, y)$ is the displacement of the point (x, y) in the y direction

The fringe pattern will be

$$\upsilon(x, y + \delta y) - \upsilon(x, y) = lp$$

By dividing both sides of equation by δy

$$\left[\frac{\upsilon(x, y + \delta y) - \upsilon(x, y)}{\delta y}\right] \cong \frac{\partial \upsilon}{\partial y} = \frac{lp}{\delta y}$$

Namely, a contour map of the partial derivatives incremented by $p / \delta p$.

Reference:

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“The Physics of Moiré Metrology”.
